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GEOMETRIC HABITS OF MIND OF MATHEMATICS TEACHERS IN THE CONTEXT OF THEIR GEOMETRY BACKGROUNDS

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ABSTRACT

This study aims to determine the geometric habits of mind of high school mathematics teachers and to examine these habits in the context of teachers' geometry backgrounds. The study implements the qualitative research method and the case study design included in it. The participants of the study are 15 mathematics teachers. Eight open-ended questions revealing geometric habits of mind and an interview form about geometry background were used as data collection tools. The data were analyzed using descriptive and content analysis methods. It was observed that the teachers who had common results regarding the use of geometric habits of mind also showed similarities in their geometry backgrounds. Between the teachers who could use geometric habits of mind in their solutions and those who could not, the results were in favor of the ones who could them.

Keywords: Geometry education, geometric habits of mind, geometry background, high school, mathematics teacher

1.0 INTRODUCTION

Mathematical thinking is the actualization of a number of different mental processes to produce solutions to problems present in daily life. Considering that mathematical thinking starts with a problem, problem-solving practices are mainly carried out in the thinking process (Henderson, 2002). A correct problem solution with a clear way of reasoning becomes a mental habit if it is applicable in different subject areas of mathematics and if it is

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internalized by the individual (Sezer, 2019). Mental habits are the ability of an individual to develop different solutions while solving a problem and to choose and apply the most appropriate solution to the problem at hand (Cuoco et al., 1996; Leikin, 2007). Mental habits are discussed in two types: ones that are present in each discipline (general) and ones specific to only one field (Cuoco et al., 1996). Mathematical habits of mind, which are evaluated within the scope of general mental habits in terms of the application area and specific to a specific field in terms of meaning, attract attention mostly as algebraic or geometric habits of the mind (Gürbüz et al., 2018; Yavuzsoy Köse & Tanışlı, 2014). Driscoll's definition of habits of mind stating that "algebra can be learned successfully if these habits are truly used as a habit" is a proof of this (Lim & Selden, 2009).

Geometric habits of mind are a productive process defined as generating new solutions and generalizing geometric ideas through geometric relationships (Bülbül, 2021b; Turan & Kurtuluş, 2021). In order to achieve this productivity, students should be left alone with any problem and thus allow them to create their own geometry knowledge (Fan et al., 2017; Gordon, 2011). The process that bridges the gap between the solutions of these problems and the mental work behind these solutions is defined as geometric habits of mind (Cuoco et al., 1996). The framework of geometric habits of mind was defined and it was stated that it had four basic components: Investigating ınvariants, balancing discovery and reflection, reasoning with relationships, and generalizing ıdeas. (Lim & Selden, 2009; Turan & Kurtuluş, 2021).

It is stated that the level of success in geometry will increase with the development of geometric habits of mind (Bülbül, 2021a; Uygan, 2016). As a matter of fact, there are studies in the literature that concluded that both students and teachers should adopt these habits (Lim & Selden, 2009; Yavuzsoy Köse & Tanışlı, 2014). Since the development of these habits that are desired to be adopted will have a positive effect on the success of the learners in many areas, it is very important for the instructors to create learning environments that develop the geometric habits of mind (Bülbül, 2016; Erşen, 2018; Uygan, 2016). Learning environments should be aimed at providing the ability to recognize the mental habits that are the source of learning outcomes (Cuoco et al., 1996). In this context, Goldenberg (1996) stated that in learning environments that support geometric thinking habits, students should be provided with a continuous and planned opportunity to experience a process of discovery.

The geometry and mathematics that will be developed in our century will be the basis of scientific developments in the next one, and the habits of mind used by scientists will be reflected in the systems that will shape our daily lives. In this sense, it is important to know the current levels of teachers, who are the determinants in the creation of a learning environment that helps to recognize the habits of mind used for the purpose of raising individuals who have foresight about a technology class that does not exist yet and prepares the ground for their use (Cuoco et al., 1996). As a matter of fact, it is known that teachers' awareness of the geometric habits of mind is effective on students' understanding of geometry (Driscoll et al., 2007). In fact, there are studies showing that the mistakes made by students and teachers are similar. It is stated that teachers' understanding of geometry is effective in helping students cope with geometry (Özen, 2015). In this case, it is crucial to determine the factors affecting the geometric habits of mind that shape the geometric thinking processes of the teachers in terms of shaping geometry teaching.

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Engaging in geometry is not just a lecture, it is a process that inadvertently settles in our memory with some familiar concepts during its routine development process. In fact, Van Hiele (1999) stated that the geometric thinking process begins with playing with shapes at a very young age. Through games, diagrams about the shapes and the relationships between them are created, and these diagrams prepare a basis for formal geometry learning (Van Hiele, 1999). In other words, geometry is automatically included in our lives at every stage of our lives (Zeybek, 2019). Altun (2018) expressed the importance of geometry in teaching by stating that there are many geometric shapes around people in their real lives and they deal with in a day-to-day basis. Thus, it can be said that geometry learning, and geometric thinking habits are not only related to the current and past geometry subjects in learning and the subjects they are related to, but also to the schemas created through life itself. Identifying the experiences that constitute the source of the mentioned schemas can be an important guide in this sense. With this point of view, by trying to determine which situations are the sources of which thinking habits, suitable learning environments can be prepared for the development of geometric thinking habits. Considering that teachers' thinking habits in geometry will be effective on students as a result of teachers' key position in student learning and their efforts to continue their teaching in accordance with their own views and beliefs (Gündüz et al., 2017; Özen Ünal & Köse, 2019), the importance of teachers' previous experiences about geometry emerges as an inevitable result. In fact, the experiences in question differentiate the teachers' perspectives on the subjects that are agreed upon in the effective teaching of geometry subjects (Zeybek, 2019). The difference in perspective shapes the teaching method, which directly affects the bond between the student and the geometric knowledge to be transferred. Aslan Tutak (2011) argued in his study that pre-service training of teachers should be handled in terms of content knowledge, supporting this view. In the current study, the subject of the geometric backgrounds of mathematics teachers, which highlights their geometry experiences, makes the findings obtained from the study valuable both in terms of student learning and the development of geometric habits of mind.

Students describe geometry subjects as difficult (Zeybek, 2019). There are many studies conducted to determine the reasons for their strain (Bayrakdar Çiftçi et al., 2013; Cansız Aktaş & Aktaş, 2012; Ergün, 2010; Okur, 2006; Özkeleş Çağlayan, 2010; Özsoy & Kemankaşlı, 2004). Although some new applications have emerged in light of the studies, the students' perception of difficulty and the problems in learning geometry could not be eliminated. For this reason, this study aims to contribute to the field by choosing geometric habits of mind in the field of geometry. It is our belief that the results of the study conducted for this purpose will also contribute to the literature. As a matter of fact, although there are studies on the characteristics that a teacher should have for effective mathematics teaching (Anthony & Walshaw, 2009; Başer & Cantürk Günhan, 2010; Kaiser & Vollstedt, 2007; Perry, 2007; Wang & Cai, 2007; Whitehurst, 2002), no study has been found examining teachers' geometric habits of mind in the context of their geometry backgrounds in the literature.

This study aims to determine the geometric habits of mind of high school mathematics teachers and to examine these habits in the context of teachers' geometry backgrounds. In this context, answers to the following questions will be sought:

1. What are the geometric habits of mind of high school mathematics teachers?

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2. What is the geometry background of the high school mathematics teachers and what are their views on this background?
3. Is there a relationship between the geometry backgrounds of high school mathematics teachers and geometric habits of mind? If so, how?

2. METHOD

2. 1 Research Design

The study uses the qualitative research method and the case study design within the scope of this method. Since the study focuses on high school mathematics teachers' geometric habits of mind and their views on geometry backgrounds, the study requires multiple instrumental case patterns, one of the case study types. The multiple instrumental case pattern purposefully determines multiple situations belonging to a chosen topic in order to best understand a situation and to reveal its different aspects (Creswell, 2013; Merriam, 2018).

2. 2 The Study Group

In qualitative research, the study group is not seen as the source of similar data from all the participants in the study, but as a deliberately created source to provide an in-depth examination of the examined situations. For this reason, it is recommended to use purposeful sampling methods that enable the collection of rich data about the research problem (Creswell, 2013; Patton, 2018). A total of 15 mathematics teachers were included in the study. These teachers were determined by using the criterion sampling method. The aim is to consider all units that meet the criteria created by the researcher or determined in accordance with a ready criterion (Creswell, 2013; Yıldırım & Şimşek, 2018). In this context, among all mathematics teachers, those working in the High school were determined as criteria. Teachers were allowed to participate in the study voluntarily. Table 1 presents the demographic characteristics of the participants.

Table 1: Demographic characteristics of the participants

Participants	Graduated high school	Bachelor's degree	Graduate degree	Type of school the teacher is employed	Experience in years
T1	Vocational high school	Faculty of science and literature	-	Anatolian high school	20
T2	Teacher training high school	Faculty of education	-	Vocational high school	23
T3	Regular high school	Faculty of education	-	Vocational high school	21
T4	Regular high school	Faculty of science and literature	-	Anatolian high school	22

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T5	Teacher training high school	Faculty of education	-	Vocational high school	13
T6	Regular high school	Faculty of science and literature	Graduate degree	Vocational high school	17
T7	Teacher training high school	Faculty of education	-	Vocational high school	11
T8	Science high school	Faculty of science and literature	-	Anatolian highschool	20
T9	Regular high school	Faculty of education	-	Anatolian high school	15
T10	Anatolian high school	Faculty of science and literature	-	Vocational high school	5
T11	Regular high school	Faculty of science and literature	-	Vocational high school	8
T12	Anatolian high school	Faculty of education	-	Vocational high school	4
T13	Teacher training high school	Faculty of education	-	Science high school	12
T14	Regular high school	Faculty of education	Ph. D.	Anatolian high school	15
T15	Regular high school	Faculty of science and literature	Graduate degree	Science high school	16

2. 3 Data Collection Tools

2. 3. 1 Open-ended question about the geometric habits of mind

By using open-ended questions, it is more likely to provide data diversity by capturing different perspectives without categories of predetermined answers (Patton, 2018). For this reason, open-ended questions revealing geometric habits of mind were used as a data collection tool in the data-collection phase of this study, which aims to comprehensively address the indicators of geometric habits of mind. The questions were obtained from the study of Tolga and Cantürk Günhan (2019). In the mentioned study, five of the questions were selected from among the questions used by Driscoll et al. (2007), and the other three were created by Tolga and Cantürk Günhan (2019) themselves. While the geometric habits of mind of secondary school mathematics teachers were determined in the study of Tolga and Cantürk Günhan (2019), in the current study, the geometric habits of mind of high school mathematics teachers were discussed in the context of their geometry history, and this study

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was also conducted with a higher number of participants. Table 2 presents the distribution of the questions according to the required geometric habits of mind.

Table 2: Habit processes in the content of open-ended questions

Geometric habits of mind	Questions	Relationship between the habit and the question
Reasoning with relationships	Number of cubes	Relationship between volume and side-length
	Cube subtraction	Relationship between the cut interface of a cube and the number of cuts
Generalizing geometric ideas	Vertex finding	Mobility of the 3rd vertex of a triangle
	Shape creation	Generalizing areas
Investigating invariants	Area creation	Invariance of areas
	Cube model	Invariance of volumes
Balancing discovery and reflection	Center of rotation	Being creative in finding the center of rotation
	Deleted triangle	Considering the side lengths in finding the original triangle

2. 3. 2 Interview form

The data of the participants' individual views on geometry backgrounds were collected through a semi-structured interview form consisting of 14 questions. The Interview Form was created by the researcher after reviewing the literature. The form developed in accordance with the purpose of the study was evaluated by two experts in the field of mathematics education and was finalized as a result of the feedback received.

2. 4 Data Collection

In order to determine the suitability of the open-ended questions and the interview form planned to be used for the study, firstly, two teachers who did not participate in the study were asked the questions as a pilot application. It was observed that the received data was overlapping with the aims of the study and it was decided to use the questions. Data collection tools were applied one-on-one with all teachers who voluntarily participated in the study. Questions about geometric habits of mind were applied in two sessions: the first four of the questions in the 1st session and the remaining four in the 2nd session. The questions in each session have content in the context of components of geometric habits of mind. Therefore, obtaining the distribution of themes determined in the first session in the second

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session confirmed the findings. The interview made in line with the interview form prepared for the teachers' geometry backgrounds was also carried out during the second session.

2. 5 Data Analysis

The obtained data were evaluated in accordance with the theoretical infrastructure of the case study, and descriptive analysis and content analysis were used together for the analysis. The analysis unit was determined as the solutions and explanations of the questions about geometric habits of mind and the opinions expressed on geometry backgrounds. The solutions of the teachers were analyzed separately by the researcher and an expert mathematics teacher in line with the indicators of each component of geometric habits of mind. For the analysis of the data, the theoretical framework of Driscoll et al. (2007) for the components of geometric habits of mind was used. Within the scope of this framework, each component was determined as a theme, and the indicators of each component were determined as sub-themes and categories. The data analyses obtained from both analyses were compared and the cases where there were differences in the direction of the comparison results were re-examined and a consensus was reached.

The same phases as the analysis of the questions related to geometric habits of mind were used to analyze the data collected from the interview in order to determine the geometry backgrounds of the mathematics teachers. The researcher and a mathematics instructor who is an expert in the field reviewed the data, and the researcher's framework was employed in the analysis of the data collected from the interview.

To explain the cause-and-effect links between each finding derived from the analyses, first the relationships between the theme and the sub-themes were established, and then these relationships were interpreted using direct quotations from the teachers' solutions.

2. 6 Validity and Reliability of the Research

Validity in qualitative research refers to the presentation of data in an objective manner. The definition of the concept of reliability suggesting that the results obtained in the research are to be obtained in a similar fashion for similar situations, contradicts the features in qualitative research, as the facts change according to each condition and that perceptions are important. Some applications, however, are recommended to assure reliability (Yıldırım & Şimşek, 2018). The evaluations were concluded in light of expert opinions in the context of these applications by referring to the expert review in this study. The study group was formed by utilizing the criterion sampling approach to try to find as many participants as possible who were suitable for the study, and the participant characteristics were stated in detail. Furthermore, the data sources, data collecting, and analysis techniques are all thoroughly discussed. The findings are expected to serve as a guide for people who are interested in the subject. The study attempted to ensure reliability by having the examinations conducted by a mathematics instructor who is an expert in his field, and the interview form of the created geometry resume was submitted to the experts for their opinion. Furthermore, the data obtained in the study are preserved so that the coding created in the analysis of the data can be examined when necessary.

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The study attempted to preserve the secrecy of information sources by identifying the volunteers as T1, T2, T3, and so on. Participants were informed about the study's content and the purpose for which the results would be used, and those who volunteered to participate were enrolled. Furthermore, because three of the open-ended questions used to determine the geometric habits of mind of mathematics teachers were borrowed from a study published in the literature, the study's owners were contacted for their consent.

3. FINDINGS

The goal of the study is to determine high school mathematics instructors' geometric habits of mind and to evaluate these habits in the context of the teachers' geometry backgrounds. For this purpose, it was discovered that the majority of the teachers had similar situations in two different questions parallel to each component, and the solutions complemented each other in the answers to the eight questions directed to the teachers. If a component indicator could not be determined in both solutions to the two problems in question, it was assumed that the teacher could not employ that component. The results of two questions comprising each component were compared and contrasted, and it was attempted to call attention to the similarities and differences between the results. Both the visuals of the question solutions and the conversations in the solution process were employed in the context of the components of each teacher's mental geometric habits, and their geometry resumes were presented under various headings in line with the dialogues during the interview.

3. 1 Findings Regarding Geometric Habits of Mind

3. 1. 1 Findings regarding the component reasoning with relationships

The questions included in the reasoning with relationships component are number of cubes and cube subtraction questions. For the number of cubes question, teachers are expected to shape the solution by establishing the relationship between the volume of the prism and the side lengths of the cubes. Although trying to establish the solution on this basis could not reach the right result, it was considered sufficient in the context of the Reasoning with Relationships component, since the reasoning method reflects reasoning with relationships. For the cube subtraction questions, they are expected to establish a relationship between the number of cuts and the number of faces of the cube.

T1 first assumed that x pieces of 2x2x2 cubes and y pieces of 7x7x7 cubes would be used and found the x and y numbers by trial and error method, but no relationship was tried to be established between the volume and the side lengths. Then, realizing his mistake, he first placed cubes of 7x7x7 dimensions and 2x2x2 cubes in the remaining space and stated that 615 cubes should be used. T1 used the reasoning with relationships component in both solutions and only a category difference occurred. Figure 1 presents T1's first and last solutions to the number of cubes question.

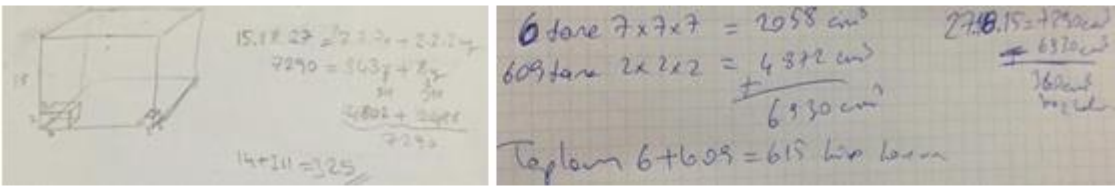

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Figure 1. First and last solutions of T1 for number of cubes question

For the cube subtraction question, T1 commented that 6 cuts should be made since the cube to be removed has 6 surfaces. Since T1 could associate the number of faces with the number of cuts, the teacher realized the order in a geometric figure and reached the Reasoning with Relationships category.

T2 used 16 cubes of 7x7x7 dimensions and 225 cubes of 2x2x2 dimensions in his placement with the least space. The problem with this solution is that T2 tried to place the cubes as if they could be pulled apart. No reflection on the Reasoning with Relationships component was detected in the solution. In the cube subtraction question, T2 stated that 6 cuts should be made and answered the question “Why 6 cuts should be made?” as "I reached 6 cuts when I started to cut the shape so that the cube in the middle could come out", that is, T2 did not use the reasoning with relationships component in his solution. Figure 2 presents the T2's solution to the cube subtraction question.

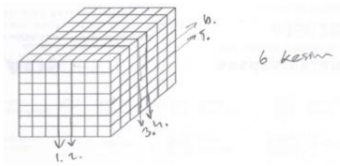

Figure 2. The solution of T2 for cube subtraction problem

The solution of T3 to the number of cubes question followed the same path as T1's first solution. For the cube subtraction question, T3 stated that he was taught that 6 cuts should be made and did not make an explanation within the scope of reasoning with relationships. T4 continued by trying to place cubes with the least space left in the solution of the number of cubes question, and by drawing the remaining space as he placed them. He completed his solution by thinking that his first attempt provided the least space. Despite the wrong answer, it can be seen that he used Reasoning with Relationships in his solution method. Figure 3 presents T4's solution.

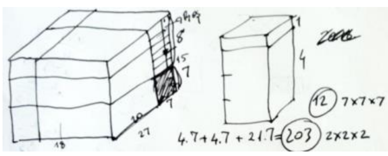

Figure 3. The solution of T4 for the number of cubes quesion

T4 made the same comment as T1 for the cube subtraction question.

T5 and T6 tried to place the cubes in the number of cubes question just as T4, but they made a right start for the solution by trying to place the cubes with 7x7x7 dimensions first. Figure 4 presents T5's solution.

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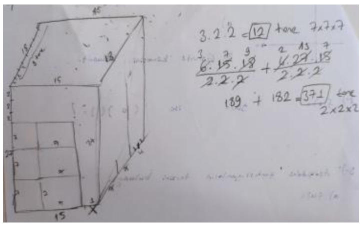

Figure 4. The solution of T4 for the number of cubes question

Both teachers reached the same solution as T2 for the question.

T7 solved the number of cubes problem the same way as T1's first solution. In the cube subtraction question, he made the same comment as T1. Since T8 started the solution by considering the relationship between the volume of the rectangular prism and the side lengths in the number of cubes question, by seeing the parts in a geometric shape, he points to the reasoning with relationships category of two geometric shapes. For the cube subtraction question, he followed the same solution as T2.

T9 stated that the highest and lowest number of cubes should be specified for the number of cubes question. He ignored that the maximum number of cubes should be used in order to have the least space. T9 followed the same path as T2 for the cube subtraction question. T10 thought the same way as T1's first solution on the number of cubes question. In the cube subtraction question, however, the Reasoning with Relationships component could not be observed, since no comment could be found. T11 started the solution by considering the relationship between the volume of the rectangular prism and the side lengths in the number of cubes question, providing reasoning with relationships. However, he reached the wrong solution by stating that he could not fully visualize the installation in his mind. For the cube subtraction question, he made the same comment as T2.

T12 started solving the number of cubes question by trying to place the cubes so that there's the least amount of space left. He first placed the cubes with dimensions of 2x2x2, after placing the cube of maximum 7x7x7 dimensions that could fit on the 27-unit edge. After this placement, he continued to leave the least amount of place. Placing the cubes as T12 did reveal the expected correct result. In this case, teacher T12's action is in the category of reasoning with relationships, suggesting the theme of focusing on parts in a single shape. The solution of T12 is presented in Figure 5.

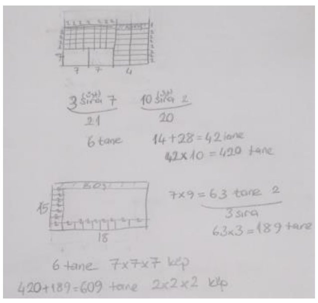

Figure 5. T12’s solution to the number of cubes question

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T12 followed the same thought process as T for the number of cubes question.

T13 reached the conclusion that 12 cubes with dimensions of 7x7x7 and 229 cubes with dimensions of 2x2x2 should be used in the number of cubes question. The Cube subtraction question was solved the same way as T2. T1 followed the same solution path as T12 for the number of cubes question. He stated that 3 cuts should be made in the cube subtraction question, but he could not use the reasoning with relationships component for it. T15 followed the same solution path as T12 for the Number of cubes question and T1 in the Cube subtraction question.

3. 1. 2 Findings regarding the component generalizing geometric ıdeas

Vertex finding and shape creation questions are included in the Generalizing Geometric Ideas component. For the vertex finding question, it is expected from the teachers that by generalizing the infinite number of moving third points of the triangle, they realize that the coordinates of the possible third point are on an ellipse, and take into account the dynamics of these points in the context of the Investigating Invariants component. Said question was also evaluated in terms of the balancing discovery and reflection component, as required by the expected generalization. Participants are expected to generalize the fields. They were asked to draw another example in option c, which has the same area as the triangle with ¼ of the area of the original square obtained in option b of the question, and thus the generalization component was tried to be revealed. In each option of the question, it is expected from them to indicate with a general expression that the desired areas will be correct for all triangles and squares.

T1 falls into the advanced generalization theme within the scope of the generalizing geometric ideas component, as it can determine the correct situation by seeing all the solution sets in the vertex finding question. In addition, since he realized that an ellipse was formed as a result of thinking about the effect of continuous motion of a point, he was able to think dynamically in terms of the Investigating Invariants component. Pointing out that the points on the y-axis are not included in the ellipse formed here, he drew attention to the conditions of being a triangle. With this explanation, it seems that he can take into account limited and extreme cases. If evaluated in the context of the balancing discovery and reflection component, it brings to mind the theme of outstanding discovery, as it states that all triangles obtained using familiar geometric concepts will form an ellipse. In the shape creation question, he determined the shapes with the desired area in each option of the question and reached a general expression about their areas. In addition, he was able to draw a different example of the triangle he drew in option b, in option c. Therefore, participant T1 was able to make advanced generalizations by determining a general rule for any geometric figure.

In the vertex finding question, T2 stated that the third point of the triangle is on a circle, saw infinity, but reached the wrong conclusion about the set and represented it with the wrong geometric figure. In other words, it is included in the transition theme in the context of generalizing geometric ideas. Reaching the circle by noticing the constant movement of the third point shows that he can think dynamically. Considering the existence of a generalization, even if it is wrong, it was concluded that familiar strategies were used in terms of the balancing discovery and reflection component since the generalization was associated with the known circle equation. The solution for T2 is presented in Figure 6.

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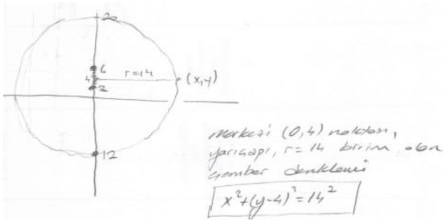

Figure 6. T2's solution to the cube subtraction question

In the shape creation question, T2 determined what was wanted in a, b, c by showing them in the drawing, but for option d, he stated that "No matter how many units we consider one side of the original square, we cannot get the desired square exactly." and was able to determine the desired situations in limited situations and took part in the transition theme in the context of the generalizing geometric ideas component.

Participants T3, T4, T6, T12, T14, and T15 sound the same solutions as T1 in both questions. The solution to the vertex finding question of T14 is presented in Figure 7, the solutions of T4 and T15 to the shape creation question are presented in Figure 8 and Figure 9.

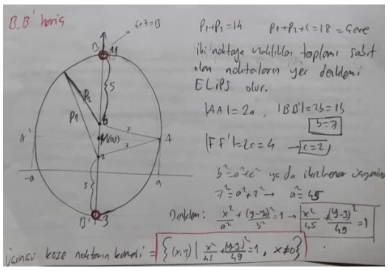

Figure 7. T14's solution to the vertex finding question

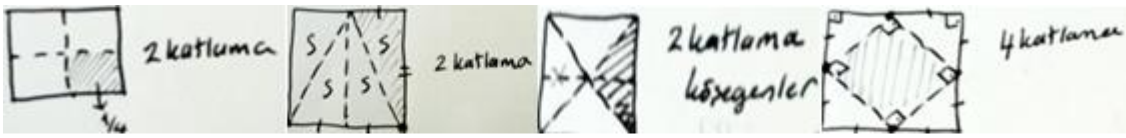

Figure 8. T4's solution to the shape creation question

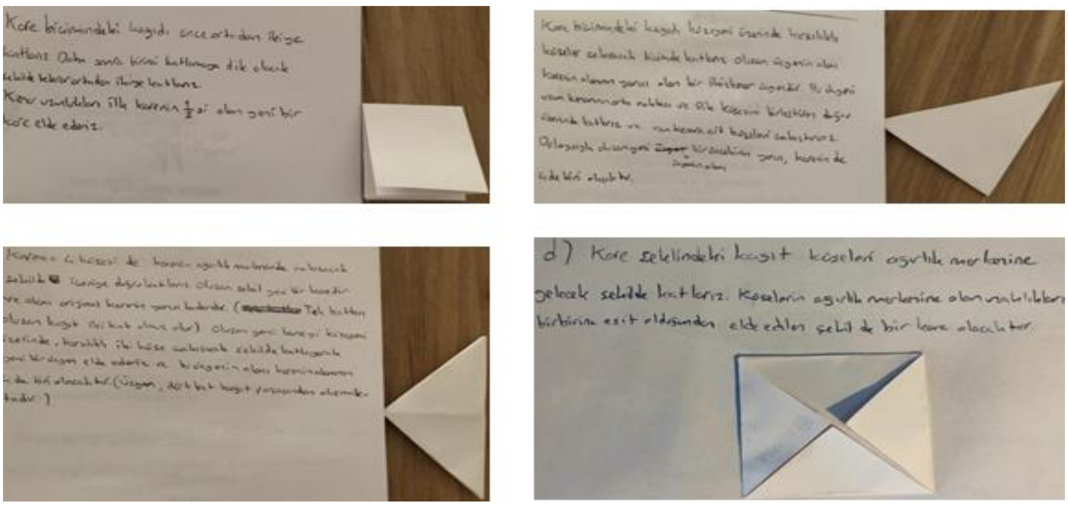

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Figure 9. T15's solution to the shape creation question

For the vertex finding question, T5 could not make any generalizations, considering the set of points that provide the equations he obtained by considering the other two sides that are not on the y-axis separately in right triangles. In the shape creation question, however, he applied the same folding as T4, and in option c, he was able to create the desired area with a different folding method. T5 teacher's solution for c is presented in Figure 10.

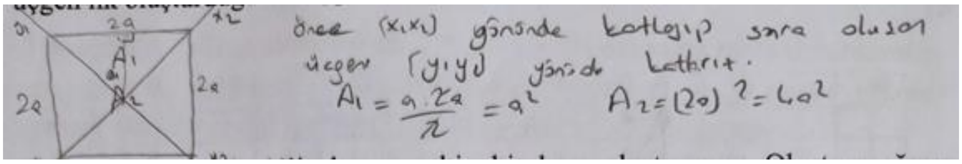

Figure 10. T5's solution to the shape creation question part c

T7 solved the vertex finding question using the same solution method as T6 but did not comment on the shape creation question. T8 thought similarly to T5 in the vertex finding question, and could not have an idea about the mobility of the point. The generalization component could not be observed clearly because the shape creation question made a fold for only the parts a and d. T9 solved the vertex finding question using the same method as T5. The generalizing geometric ideas component could not be clearly observed, since only part a of the shape creation question was folded correctly. T10 used the same solution as T9 in the vertex finding question. In the shape creation question, however, the generalizing geometric ideas component could not be observed, since it could not explain the desired with a general expression. T11 thought in the same way as T10 in the vertex finding question, but since he could only fold correctly in a part of the shape creation question, no generalization could be observed. T13 could not reach any generalization in the vertex finding question, created the triangle that was requested to be formed in part b of the shape creation question, but could not create a similar one in part c, that is, he could not think of a situation similar to it. In other options, he was able to determine the desired areas with a general expression. However, it can be seen that the use of generalization is not clear since it does not fulfill all the expectations in the question.

3. 1. 3 Findings regarding the component investigating invariants

These are area creation and cube model questions within the scope of the Investigating Invariants component. In the area creation question, the teachers are expected to realize that the area will not change by shifting the apex of the triangle to be parallel to the base while constructing triangles with the same area, and to be able to identify the triangle with the largest perimeter among the triangles with the same area within the dotted grid. In the cube model question, by applying different transformations to the given cube models and considering the cube numbers in the models created with the number of cubes used, it is expected to obtain the desired cube models in three different orders and to make correct comments about the volumes of the new cube models created in the meantime.

While T1 drew the triangles whose areas were given in part a of the area formation question, he drew only right triangles, and in part b, triangles with an area of 1 square unit in three different styles as right, isosceles, and obtuse triangles. In part c, he pointed out the theme of

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checking the evidence of effects in the context of the Investigating Invariants component, noticing that when the vertex of the triangle whose base is 1 unit changes, there is no change in the area. In part d, he stated that he can obtain triangles with the same area by changing the vertex so that the base is the same. He obtained the desired models by placing the appropriate cube models in the cube model question in the correct order. To the researcher's question “What is your interpretation of the volume in the new models formed?”, he replied “The volumes of the first and second models are the same. The volume of the third model is different because of the number of cubes used.” That is, his solution reflected the theme of checking for evidence of effects.

T2 drew triangles that accept the positive integer divisors of the areas given in part a of the area creation question as base and height and stated that various triangles can be formed by shifting the vertex of the triangles formed in part b on a line parallel to their bases. In other words, realizing what has not changed with a transformation implemented within the scope of Investigating Invariants, he expressed his reasoning, pointing to the theme of checking the evidence of effects. The solution of T2 for part b of the question is presented in Figure 11.

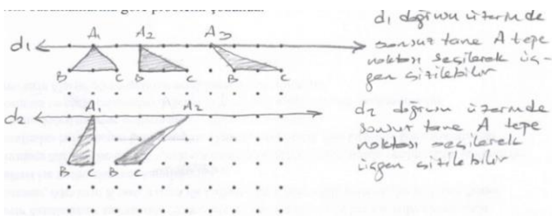

Figure 11. T2’s solution to the area creation question part b

In part c of the problem, T2 thought that the triangle he drew in part b, the triangle with the largest circumference, is obtained by shifting the vertex by 9 units. He added that he could obtain an infinite number of triangles by shifting the vertex. With this statement, he confirmed that he could realize what would not change with a transformation implemented, being conscious of the reason. However, it has been overlooked that the base and height can be taken as 9 units at the most. In the cube model question, he used the same solution method as T1.

While drawing triangles with desired areas in the area creation question, T3 focused on a single base and height, determined all triangles with an area of 1 square unit in part b, and said "the vertex of the triangle can be shifted more parallel to the base". In part c, he stated that for the perimeter of the triangle with the largest area, the vertex should be shifted so that the hypotenuse length is the largest. Realizing that there was no change in the area by changing the apex of the triangle with a base of 1 unit suggested the theme of controlling the evidence of effects. In the D part and the cube model question, he used the same solution as T1.

T4 ignored obtuse triangles in the c part of the area creation question and failed to realize that the area would not change with the change of the vertex. His thought process was similar to T3 in part d of the question. To the researcher's question: "Are there any other triangles that can be drawn in part c?" He replied, "I ignored obtuse triangles, because shifting the vertex

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does not change the area". This answer is proof of being conscious of invariants with any transformation applied. He did the same solution as T2 in the cube model question.

T5 thought the same way as T2 in parts a and d of the area creation question. In part b, he focused on triangles with a base of 2 units and ignored those with a base of 1 unit. He realized that the area would not change by shifting the vertex on the line parallel to the base. In part c, he stated that among the triangles he drew in part b, the triangle with the largest circumference was obtained by shifting the vertex by 9 units, and he highlighted the category of feeling that everything has not changed in a transformation applied in his solution. In the cube model question, he was able to determine the models by using the correct cubes except for the first model. It was observed that since he could not make any comments about the volumes of the new cube models, he could not notice the unchanging features. T6 followed the same path with T1 in a, b, and c of the area creation question, with T2 in the d part, and with T5 in the cube model question. T7 couldn't find any solution to the area creation question. In the cube model question, he was able to identify the 2nd and 3rd models, but could not make any comments on their volumes. The Investigating Invariants component could not be observed in the solution of T7.

T8 formed triangles by thinking that in a part of the area creation question, the product of the base and the height should be twice the desired area and that the positive integer divisors of the product and these integer divisors should be at most 9 units. In part B, he considered only right triangles with a base of 1 and a height of 2 units. In other words, he could not think that there would be no change in the field by changing the vertex. In part c, obtuse triangles were not taken into account. In part d, he stated that the triangle area, whose base and height are 9 units each, is the largest. He could not answer the question "Are there any other triangles with this area?" with a meaningful justification. In the cube model question, he determined the 3rd model incorrectly, could not determine the other two models, and could not comment on the volume. T8 did not use the investigating invariants component in the solution.

While T9 only considered the drawings specified in part a of the area creation question as right triangles, he could not find any solution to the other parts of the questions. He could not find any solution to the cube model question either. In the part a of the area creation question, T10 made the desired drawings for all the triangles except the triangle whose area is 3/2 square units. He could not evaluate the situation that the area would not change with the change of the vertex in part b, and he could not make any comments for part c. Although he stated that the triangle area with a base and a height of 9 units in part d is the largest, he could not consider the existence of other triangles with the same area. In the cube model question, he tried to create the desired cube models without paying attention to the cube numbers in the given models, and could not make a prediction about the volume because he did not focus on the cube numbers. The investigating invariants component could not be observed in the solution of T10.

T11 used the same solution as T6 in part a of the area creation question. In part b, he did not realize that there would be no change in the area by changing the apex on the line parallel to the base. In part C, he could not find the desired triangle. Choices b and c show that T11 cannot reflect the investigating invariants component to its solutions. He shared this thought with T10 in part d and confirmed his position regarding the Investigating Invariants component. In the cube model question, he only created the second model correctly and

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stated that he created it by imagining it in his mind. From this, it can be concluded that he did not focus on cube numbers. T12 has drawn only right triangles with the areas given in part a of the area creation question. He thought similarly T5 in part b. He stated that the areas of the triangles will not change by shifting the vertex of the triangles on the line parallel to the base. In part D, he said that by shifting the vertex, triangles with the same area can be obtained. In the cube model question, he thought the same as T4.

T13 followed the same solution path as T2 in parts a and b of the area creation question. He could not consider the obtuse triangle in part c. He calculated the largest area correctly in part d, but he could not notice the unchanging situations since he could only draw for some cases that the area would not change when the vertex was shifted. In the cube model question, only the third model was able to form correctly. Here, it is understood that he ignored the cube numbers and used 8 cubes instead of 7 cubes while creating the first model. Figure 12 presents T13's solution.

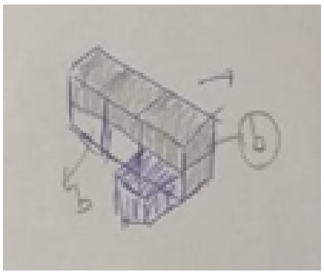

Figure 12. T13’s solution to the 7th question’s 1st model

T14 determined all of the triangles whose areas were given in a part of the area creation question and, unlike the other participants; he determined more triangles by evaluating the base and height outside the horizontal axis, with all the corners of the triangles above the points. In part b, he drew all the triangles whose area is 1 unit square, and when he shifted the apex of the triangle parallel to the base, he noticed the stability of the area and showed that he was conscious of the unchanging ones. In part c, he took obtuse triangles are into account. In part d, he showed that he could reflect the Investigating Invariants component to his solution by using the statement "We can draw different triangles with the same area by shifting the vertex". In the cube model question, he made the same volume interpretation as T4. Figure 13 presents the solution.

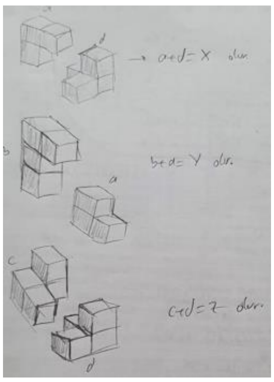

Figure 13. T14’s solution to the cube model question

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T15 thought the same was as T14 in area creation and cube model questions.

3. 1. 4 Findings regarding the component balancing discovery and reflection

Center of rotation and deleted triangle questions are included in the balancing discovery and reflection component. In part b of the center of rotation question, teachers are expected to find the centers of rotation in P center of rotation, taking into account that neither the distance from the rotating line segment AB nor the distance of the center of rotation they find to the rotating line segment changes. In the deleted triangle question, teachers are expected to be able to describe the steps applied to find the deleted ABC triangle.

T1 stated that in the center of rotation question, a circle with center P will be formed. In part b, the balancing discovery and reflection component could not be observed, since the centers of rotation could not be found. Couldn't find any solution to deleted triangle question. He showed the shape formed in the T2 center of rotation question by drawing. During the rotation, he could not see the constancy of the distance from the line segment AB to point P. The balancing discovery and reflection component could not be detected with clear indicators, since he could not determine the appropriate centers of rotation in part B. He could not comment on the constancy of the distance between the center of rotation and the rotating line. He found the right triangle by making additional drawings in accordance with the steps to be applied in the deleted triangle question. The balancing discovery and reflection component was observable because it was able to determine what the final state looked like by correctly identifying the intermediate steps. However, due to the determination in the center of rotation question, it has been seen that the way of thinking about the balancing discovery and reflection component is unclear.

T3 stated that a ring was formed in the center of rotation question and correctly determined all the rotation centers in part b. Since it reached the result in accordance with the steps to be applied, it made us think about the category of using auxiliary intermediate steps to reach the goal. He stated that the distance between the rotating line segment and the rotation centers and the distance during the rotation of the AB line segment around the P point do not change. He came up with the same solution as T2 in the deleted triangle question. T4, T6, T14, and T15 applied the same solution method as T3 in both questions.

In the center of rotation question, T5 thought that a polygon with an infinite number of sides would form and could not interpret the distance during rotation. He could not correctly identify most of the centers of rotation. In the deleted triangle question, he drew the deleted triangle correctly by making use of the similarity. However, due to inaccuracies in the center of rotation question, the way of thinking about the balancing discovery and reflection component was unclear. T7 showed by drawing that a ring was formed in the center of rotation question. In part B, however, he could not determine the majority of the rotation centers. In his solution, he referred to the constancy of the distance during rotation. He drew the deleted triangle incorrectly by misinterpreting the steps to be applied in the deleted triangle question. The balancing discovery and reflection component could not be observed clearly. Figure 14 presents the solution and explanation of T7's deleted triangle question.

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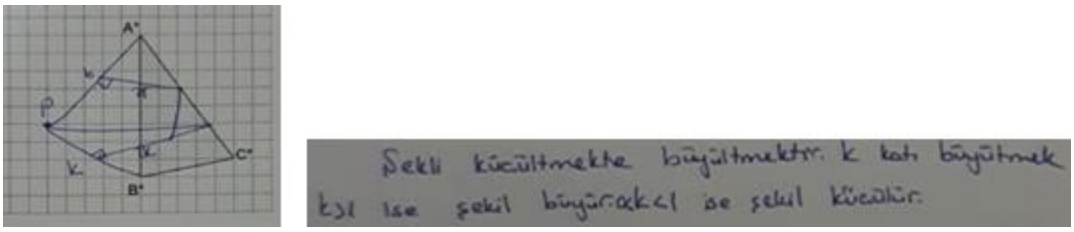

Figure 14. T7's solution to the deleted triangle question and his comment

In the center of rotation question, T8 stated that a semi-cylindrical structure would be formed. In part b, the participant determined all the centers of rotation as a point on the left line segments. He stated that the distance between the rotation centers and the line segment is "fixed because they are symmetrical". T8 could not vision the rotation of the right line segment so that it overlaps the left one. The balancing discovery and reflection component could not be observed because no comments were found on the deleted triangle question. In the center of rotation question, T9 wrote the same solution as T6. However, balancing discovery and reflection component could not be observed because it could not provide any solution to the deleted triangle question. T10 displayed the same way of thinking as T1 in both questions. T11 used the same solution as T8 in both questions. T12 applied the same solution as T7 in the center of rotation question, but could not find any solution in the deleted triangle question. T13 thought in the same way as T3 in the center of rotation question and drew a wrong triangle by incorrectly applying the intermediate steps specified in the deleted triangle question, the final purposes sub-theme that stood out within the scope of the balancing discovery and reflection component was not clearly observed.

3. 2 Findings Regarding Geometric Backgrounds

In line with the data of geometry backgrounds obtained through interviews with teachers, a framework containing themes, categories, and codes was created. The teachers who created the said framework and the determined codes are listed in Table 3.

Table 3: **Themes, categories, and codes of geometric backgrounds**

Theme	Category	Code	Participants who emphasised the code
Applications of the concept of geometry	Real-life associations	Using the Pythagorean theorem	T1, T3, T4, T6, T12, T14, T15
		In engineering and architecture	T1, T4, T6, T12, T15
		Examples of Fibonacci numbers in nature, golden ratio	T1, T2, T3, T4, T5, T6, T12, T14, T15
		Symmetry and similarity	T1, T2, T11
		Using daily objects	T1, T5, T7, T8, T9, T10, T11, T12, T13
		Sports activities	T3, T14

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	Associations made on trips	Sine quadrant, tools proving the Pythagorean theorem	T1
		City plans of ruins, motifs in mosques and historical buildings	T1, T2, T3, T4, T5, T6, T9, T7, T10, T11, T12, T13, T14, T15
		Cubical shapes in tables	T1, T3, T4, T6, T12, T14, T15
	Event attendance	No	T5, T7, T8, T9, T10, T11, T13
		In-service training (geogebra)	T2, T4, T6, T12
		Workshop	T1, T3, T12, T14, T15
Geometry education past	Teaching style	Evidence-based	-
		Rule-dependent	T2, T3, T5, T6, T7, T8, T9, T10, T11, T13
		Rule and evidence together	T1, T4, T12, T14, T15
	Geometry class during high school education	No	T2
		Yes – with presentation	T3, T4, T5, T6, T7, T8, T9, T10, T11, T13, T14, T15
		Yes – with discovery and presentation	T1, T12
	Geometry class during undergraguate school	Yes – with presentation	T2, T3, T4, T5, T6, T7, T8, T9, T10, T11, T13, T15
		Yes – with discovery and presentation	T1, T12, T14
	Graduate school (in the field)	No	T1, T2, T3, T5, T6, T7, T8, T9, T10, T11, T12, T13
		Yes -finished	T4, T6, T14, T15
		Yes - unfinished	T1, T3, T12
	Making geometric drawings	No	T2, T5, T8, T9
		Yes – using compass, protractor, ruler	T1, T3, T4, T6, T7, T10, T11, T12, T13, T14, T15
	Using geometry software	No	T1, T2, T3, T4, T5, T6, T7, T8, T9, T10, T11, T13, T14, T15
		Yes	T12 (maple)
	Using geometric	No	T2, T5, T7, T9
		Yes- at a basic level	T8, T10, T11, T13

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	evidence	Yes- at an advanced level	T1, T3, T4, T6, T12, T14, T15
Geometry teaching preferences	Improvisation-lecture	Lecture	T1, T2, T7, T8, T9, T10, T11
		Improvisation	-
		Lecture + improvisation	T3, T4, T5, T6, T12, T13, T14, T15
	Course	Required	-
		Elective	T2, T5, T7, T8, T9, T10, T11, T13
		Required and elective	T1, T3, T4, T6, T12, T14, T15
	Teaching style	Evidence-based	T5, T7
		Rule-dependent	T8, T9, T10, T11, T13
		Rule and evidence together	T1, T2, T3, T4, T6, T12, T14, T15
Geometry teaching past	Active teaching style	Evidence-based	-
		Rule-dependent	T5, T7, T8, T9, T10, T11, T13
		Rule and evidence together	T1, T2, T3, T4, T6, T12, T14, T15
	Using geometry software	No	T2, T5, T7, T8, T9, T10, T11
		Yes (geogebra)	T1, T3, T4, T6, T12, T13, T14, T15
		Yes (skratch)	T1
		Yes (cabri)	T12, T14
		Yes (derive 6)	T14

When Table 3 is examined, it can be observed that the codes were formed with the answers received in the interviews with the teachers, the categories with similar codes, and the themes with the similar ones of the categories. The real-life situation that most of the teachers associate geometry with is the natural equivalent of the golden ratio and Fibonacci numbers, the situations that which all of the participants except T8 pay attention to geometry on trips are the motifs in mosques and historical buildings and city plans put up in ruins. Teachers participating in the study only attended geogebra training in terms of in-service training. While all of the teachers stated that they follow a publication related to geometry, T1, T3, T4, T6, T12, T14, and T15 stated that they also follow a journal that is about mathematics but also occasionally includes geometry. Participated activities are stated as geogebra in-service training and workshops on geometry. They evaluated the activities as beneficial in terms of sharing information, gaining three-dimensional thinking skills, and learning easy teaching methods. Most of them have learned geometry only as rule-dependent, and their preference in teaching is to deal with rules and proofs together. They generally maintain these preferences in their active education. All teachers except T2 took geometry as a separate subject in high school. It is seen that the presentation method is used extensively in both high school and

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undergraduate education. T14 received Ph.D. education, T4, T6, and T15 postgraduate education, T1, T3, and T12 discontinued their graduate education, while other teachers did not receive any postgraduate education at all. Except for T12, those who did not use any geometric software in their education life attributed this to the fact that these programs were not very common at that time. The majority of teachers do not use these softwares intensively in their teaching lives. The number of teachers using programs other than geogebra is also very low.

4. RESULTS AND DISCUSSION

When the obtained findings were handled separately in the context of the components of geometric habits of mind, some common results were reached. Within the scope of the reasoning with relationships component, it was seen that most of the teachers could not do reasoning with relationships. It was determined that a small number of participants who could do reasoning with relationships were included in the sub-theme of focusing on parts in a single shape. This result coincides with Güler's (2016) conclusion that mathematics teachers have deficiencies in reasoning with relationships while teaching geometry subjects. It was observed that all of the teachers could not use their special reasoning skills within the scope of reasoning with relationships. The result of Özen's (2015) study that mathematics teachers lack knowledge about the sub-theme of using special reasoning skills supports the study. The result of Yavuzsoy Köse and Tanışlı (2014) arguing that the class candidates participating in the study could not use their special reasoning skills at the expected level is also parallel with the current study. The majority of teachers who can generalize geometric ideas can make advanced generalizations. Within the scope of the Investigating Invariants component, it was seen that the dynamic thinking teachers were also included in the sub-theme of checking the evidence of the effects. In addition, all of the teachers who can think dynamically, except for only one, were included in the categories of both thinking about the continuous movement of the point and considering extreme and limited situations. In the context of the balancing discovery and reflection component, it was determined that all of the teachers in the sub-theme of outstanding discovery used familiar strategies. Within the scope of this component, it was determined that generally, teachers were not included in the sub-theme of outstanding final goals. This finding coincides with the result of Özen (2015) arguing in his study that mathematics teachers lack knowledge in both sub-themes of exploration and reflection. Similarly, the conclusion that middle school mathematics teachers who participated in the studies of Tolga and Cantürk Günhan (2019) reached the most difficulty in the question containing the outstanding final goals sub-theme also supports this result. In addition, it was seen that the teachers who were not in the sub-theme of the final goals that stood out were not included in the sub-themes of the Reasoning with Relationships component. This result is consistent with the conclusion in Bülbül's (2021b) study stating that exploration and reflection and reasoning with relationships support each other. It was determined that the teachers who could not use the other three components except reasoning with relationships were the same teachers, and it was seen that these teachers were among the teachers who could not use the reasoning with relationships component.

When the results related to the geometry background were examined, it was seen that some of the results were valid for all of the teachers. They did not characterize the concept of geometry as just a lesson, but considered the geometrical structures of the objects in

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reasoning with relationships and the maximum efficiency of the objects they use, with their equivalents in real-life situations. This result is consistent with Güler's (2016) conclusion that mathematics teachers aim to contribute to daily life and Reasoning with Relationships and daily life in teaching geometry. Similarly, Wang & Cai's (2007) study with teachers in the USA supports the conclusion of the study, as teachers think of mathematics as knowledge related to real life and a tool for solving real-life problems. It was concluded that if geometry was an elective course, they would have chosen geometry. They stated that they did not follow a publication covering just geometry. They are of the opinion that publications should not be bogged down in details, simplified, not only theoretical, but also supported by plenty of visuals and examples, and presented in relation to real life. While they stated that they did not take a separate course that provides the ability to find proof in their undergraduate education, and they did proof studies in other courses, they took separate geometry courses mainly based on the presentation method. This result parallels the conclusion of Stipek et al. (2000) that mostly creativity and independence are not taken into account in mathematics teaching and the lessons are taught by adhering to the procedures. All participating teachers think that the use of geometry software is necessary, but they differ on their active use. The result of the studies of Çiftçi & Tatar (2015) and Yazlık (2019) argue that teachers find the use of information and communication technologies useful, supporting this study. They think that geometry cannot be learned spontaneously, or just by improvisation. They associate this situation with the fact that geometry should be learned cumulatively within a routine, this way, it will be handled as a whole and geometric concepts cannot be noticed. This result coincides with the conclusion that Perry (2007) reached in line with the opinions of teachers in Australia, that teachers needed routines and that children's mathematics-learning becomes easier with the stability provided by routines. The fact that among the participants, only the teacher with the lowest experience did not use geometry software in their education life, coincides with Yazlık's (2019) conclusion that the pre-service teachers lack experience related to technology. There is no teacher who argues that all geometry teaching should be taught within the scope of a compulsory course. In addition, no teacher has learned geometry only on the basis of proof and does not teach only on the basis of proof. The preferences of the majority of teachers regarding teaching geometry and their practices in their active teaching lives are parallel. The parallelism between the preferred method in teaching and the actively-used method is consistent with the result of Stipek et al.'s (2000) study.

When the results for the components of the geometric habits of mind were evaluated holistically in the context of the categories and codes of the geometry backgrounds of the teachers, some results in relation to each other emerged. It was observed that all of the teachers who can make advanced generalizations can also visualize the effect of the continuous movement of the point and think dynamically by noticing limited and extreme situations; It was observed that by evaluating unchanging situations with applied transformations, they can check the evidence of effects and explore a situation through familiar strategies. This result parallels the result of Özen (2015) and Tolga and Cantürk Günhan (2019). All of these teachers are male. This result contradicts Oral and İlhan's (2012) study, which includes the conclusion that gender is not effective on pre-service teachers' geometric thinking levels. The vast majority are teachers with a working year of 15 years or more and among all participants are teachers with postgraduate studies. This result is consistent with the result of Çakmak and Güler's (2014) study that the relationship between pre-service teachers' ages and their geometric thinking levels is a positive one. Looking at

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their geometry backgrounds, it was seen that they were able to evaluate geometry outside of general definitions and that they could relate more to real-life situations by looking at their surroundings with geometry awareness. This is in line with the conclusion of Anthony & Walshaw's (2009) study arguing that effective mathematics should connect with real-life and help students establish it. It was concluded that all of these teachers, unlike the others, followed a journal about mathematics in which geometry was also occasionally included. Most of them did not use geometry software in their learning lives. They associated this situation with the level of development of technology in their past. From this explanation, it can be concluded that the preference of using software is independent of individual conditions. As a matter of fact, all of them actively used geogebra in the teaching, which they use according to their own preferences. There are even those who use other software programs in addition to geogebra. This is consistent with Bülbül and Güven's (2019) study, which includes the result that the use of software positively affects the use of geometric thinking habits, and with the result of Anthony & Walshaw's (2009) study arguing that effective teachers should follow and use technological developments. It was determined that they learned geometry as rule-dependent or few proofs. However, these teachers like to use proofs and work on advanced ones. Furthermore, their preferences in geometry teaching are to deal with proofs and rules as a whole, on the grounds that they increase permanence. They do this in their active teaching lives as much as possible. These results show that these teachers showed interest in geometry from the past, and their interest remains alive no matter what their learning styles are. As a matter of fact, they mostly try to teach the geometry skills they learned as Rule-dependent, integrated with proof in their teaching life, and they work on advanced proofs. This relationship about geometric proof was found in the study of Akkan et al., (2018). It found that mathematics teachers with a higher level of geometric thinking had a better ability to verify relations. The ability to prove in Senk's (1989) study and the positive relationship between the level of thinking of geometry and Van Hiele geometry also supports this study. They think that it would be beneficial to carry out geometry teaching as compulsory for teaching basic skills and then as optional. In addition to the identified similarities, it was observed that these teachers differed in the use of the reasoning with relationships component and the final aims sub-theme that stood out within the scope of the balancing discovery and reflection component. In fact, it has been determined that the teachers who make up the sub-theme of the last goals that stand out are the teachers who have completed their postgraduate education.

There are teachers who cannot fully use the components of the geometric habits of mind. It was determined that these teachers were similar to each other in terms of some common results in the context of their geometry backgrounds. So much so that none of them evaluated the real-life situations that they associate with geometry together with a geometry concept or theorem nor made general explanations. In addition, it was observed that they did not participate in any activity related to geometry. They learned geometry as Rule-Dependent. In teaching, their preferences are either only proof or only Rule-dependent; however, it was determined that all of them also carried out Rule-dependent teaching. The view of all is that the geometry course should be elective. Some of these teachers, who stated that they did not use geometric proofs, explained that they implemented well-known proofs at a basic level. In the literature, there are studies that support this result, which includes Van Hiele's, arguing that visual proof activities affect the level of geometric thinking positively (Polat et al., 2019; Senk, 1989). In addition, it was determined that none of them used the geometric software

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actively and continuously. They did not receive any postgraduate education and did not take any initiative in this regard.

Between the teachers who could use geometric habits of mind clearly in their solutions and those who could not use it, a result in favor of the teachers who could use the habits was observed between participating in various activities related to geometry, following a publication, and being interested in the postgraduate education. This result is supported by the result of Akkan et al., (2015) arguing in their study that teachers with high geometry background scores also have high geometry thinking levels. It also shows parallelism with the result in Özen's (2015) study stating that the mental geometric habits of teachers develop through lesson studies, in which geometric thinking can be developed. Similarly, the learning environment allows the use of geometric habits of mind. Bülbül (2021a) argued that geometric habits of mind will be acquired when faced with various geometry problems, supporting the current study. Taking geometry as a separate course in high school and undergraduate education, geometric drawings, being interested in drawing, the difference in the type of faculty during undergraduate graduation, or school type at graduation did not cause any difference in terms of both geometric habits of mind and geometry background. This result is in line with Oral and İlhan's (2012) study. Whose conclusion is that the geometric thinking levels of pre-service mathematics teachers are independent of the type of high school they graduated from. However, it contradicts the conclusion of Akay and Kurtuluş (2017).

In Akkan et al., (2018) study, it was found that mathematics teachers who graduated from faculties other than education had a substantial lack of knowledge about geometry and skills to prove visual theorems. Bülbül (2016) stated that a geometry course in undergraduate education that supports geometric thinking skills positively affects prospective teachers' geometric habits of mind. Both contradict the previously mentioned notion. The results of Napitupulu's (2001) study in which geometric drawing using a compass and non-measured ruler affect the level of geometric thinking, and the conclusion of Idris (2007), that preliminary geometric drawing studies have a positive effect on geometric thinking and success, do not support the study.

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