

## INVESTIGATING ARTIFICIAL INTELLIGENCE TECHNOLOGIES IN GENERATING LVAR

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### ABSTRACT

The current study emphasizes the significance of digital media and Artificial Intelligence (AI) in developing innovative approaches to concept introduction. The study centers on an educational experiment aimed at investigating AI technologies, including ChatGPT, Leonardo.AI, Lumen5.AI, and Pictory.AI, in the creation of Linking Visual Active Representations. This involves the integration of verbal text with both virtual static content and interactive video elements. Additionally, the study introduces the revised Bloom's taxonomy, which acts as a framework for evaluating the educational experiment carried out with these AI tools. An analysis of a dialogue generated by ChatGPT is conducted through the lens of Bloom's theory, resulting in the finding that ChatGPT not only applies Bloom's framework but also incorporates various theoretical perspectives from didactics, pedagogy, and psychology to develop quizzes and dialogues. The extensive educational experiment will facilitate the extraction of insights regarding how ChatGPT utilizes theoretical frameworks to formulate questions and foster subsequent discussions among students and educators, stemming from a series of increasingly complex prompts.

**Index Terms-** Artificial Intelligence (AI) technologies, Linking Visual Active Representations, The revised Bloom's taxonomy.

### 1.0 INTRODUCTION

Digital instruction refers to the practice of teaching using digital media. Digital media are those that provide information in a digital format of representations, which can be perceived via the faculties of vision, auditory perception and tactile sensation (e.g., websites, hypertexts, video games, 3D graphics, electronic books). Accessing mathematical concepts through digital media offers different methods by which these concepts can be created and understood by students. Artificial Intelligence (AI), a term introduced by John McCarthy, an emeritus professor at Stanford University, in 1955, is defined as "*the science and engineering of creating intelligent machines*" (Manning, 2020; Rapaport, 2020). Furthermore, McCarthy and his colleagues later proposed the idea that every element of learning or any characteristic of intelligence can theoretically be articulated with such precision that a machine could be designed to replicate it (McCarthy et al., 1955).

“[...] *the conjecture that every aspect of learning or any other feature of intelligence can in principle be so precisely described that a machine can be made to simulate it*” (McCarthy et al., 1955).

The origins of educational artificial intelligence can be linked to the contributions of psychologists Sidney Pressey, who served as a professor at Ohio State University during the 1920s, and Burrhus Frederic Skinner, a professor at Harvard University from 1948 to 1974. Both scholars investigated foundational ideas concerning instructional devices and personalized learning. In his publication "Teaching Machines" Skinner (1958) emphasizes, "*Like a good tutor, the machine presents just that material for which the student is ready....*" (p. 971). Artificial Intelligence (AI) is, therefore, a term that encompasses a range of technologies, including machine learning, natural language processing, optimization, knowledge-based systems, and more. The definition of AI is referenced in numerous articles. According to the European Commission (European Commission's Communication on AI, High-Level Expert Group on Artificial Intelligence, Rossi et al., 2018):

*"Artificial intelligence (AI) refers to systems that display intelligent behaviour by analysing their environment and taking actions – with some degree of autonomy – to achieve specific goals. AI-based systems can be purely software-based, acting in the virtual world (e.g. voice assistants, image analysis software, search engines, speech and face recognition systems) or AI can be embedded in hardware devices (e.g. advanced robots, autonomous cars, drones or Internet of Things applications)." (p.1)*

According to Coursera (website [1]), AI refers to computer systems capable of performing complex tasks that only a human could do, such as reasoning, decision-making, or problem-solving. Additionally, according to the European Council (website [2])

*Artificial intelligence (AI) is the use of digital technology to create systems capable of performing tasks commonly thought to require human intelligence. AI is not a new technology. Some AI technologies have been around for decades, but the advances in computer power, the availability of large quantities of data and new software have led to major breakthroughs in a short amount of time.*

Through artificial intelligence and computational systems capable of performing tasks that inherently require human intelligence, there is the potential for speech recognition, decision-making, and the identification of patterns and rules. In general, I could consider that:

*Artificial Intelligence (AI) is a term that encompasses a range of technologies, including machine learning, natural language processing, and robotics, aimed at simulating / imitating the cognitive processes of humans such as reasoning, decision-making and problem-solving.*

AI brings together scientific theories and techniques aimed at simulating the cognitive functions of a human being. Despite these advancements, current AI systems face limitations, including challenges in achieving higher cognitive functions, generating goal-oriented video continuations, developing consciousness or moral codes, and transferring learned knowledge effectively. After reviewing the discourse among experts in the field (e.g., Constantinos Daskalakis, 2023, Real Podcast, website [3]), I have reached the following conclusions: (i) artificial intelligence is unable to perform higher cognitive functions in a clearly defined manner, (ii) it lacks the ability to generate a video continuation with a specific objective, (iii) it has not developed consciousness or a moral framework, (iv) its proficiency in Mathematics

is debatable, and (v) it has not demonstrated the ability to translate concepts it has not thoroughly mastered into a mental function with simulation capabilities.

Digital media and artificial intelligence have made a strong entry into the educational field, opening up new prospects for improving the quality of educational work and the evaluation of school units (e.g., Anastasiades, 2023). The World Economic Forum's report states that the primary goal for us as educators is to help students achieve their educational objectives (website [4, 5]). Regarding knowledge, our main aim as educators is to promote digital literacy and literacy in matters related to AI. In a broader context, adhering to the framework of the seven principles for artificial intelligence in education as outlined by the World Economic Forum, it is essential to assess the influence of AI on educational practices, foster academic integrity, ensure adherence to current regulations, comprehend the advantages of AI while mitigating associated risks, and preserve human oversight in decision-making processes.



**Figure 1:** Seven Principles for AI in Education (World Economic Forum, website [4])

Specifically, ChatGPT (*Chat Generative Pre-Trained Transformer*) (<https://openai.com/index/chatgpt/>), widely known since 2022, is considered a natural language processing tool and an AI program that has been the subject of numerous studies (e.g., Ipek et al., 2023). Some studies have expressed concerns about its introduction into education, while others highlight its potential benefits as well as possible risks (Punie, & Redecker, 2017). Generally, Sabzalieva and Valentini (2023) in their extensive study on ChatGPT describe it as a language model (chatbot) that enables people to interact with a computer naturally and conversationally, generating authentic results.

*ChatGPT uses natural language processing to learn from Internet data, providing users with artificial intelligence-based written answers to questions or prompts (Sabzalieva & Valentini, 2023, p.5)*

ChatGPT responds to user queries in the form of prompts. We will next examine how artificial intelligence and digital media influence the educational process and the evaluation of educational institutions. Additionally, we will explore ChatGPT's role in gathering information with speed and accuracy, acting as a "collector" (/ functioning in the capacity of an "angler" or a "harvester") navigating through unknown pathways on the internet. In my question what is ChatGPT's opinion with regard to AI, ChatGPT wrote (see also European Commission, 2022):

“As an AI, I don't have personal opinions or feelings. However, I can provide a balanced perspective on AI based on the information and discussions surrounding the topic. AI is a powerful tool with the potential to transform various aspects of society, from healthcare and education to business and entertainment. It offers opportunities for innovation, efficiency, and problem-solving at scales previously unimaginable. For example, AI can analyze vast amounts of data quickly, improve decision-making processes, and even assist in creating new technologies. However, AI also presents challenges and risks. Concerns about job displacement, privacy, security, and ethical issues are significant. There's ongoing debate about how to ensure that AI is developed and used responsibly, with considerations for fairness, transparency, and accountability”.

## 2.0 BLOOM’S REVISED TAXONOMY

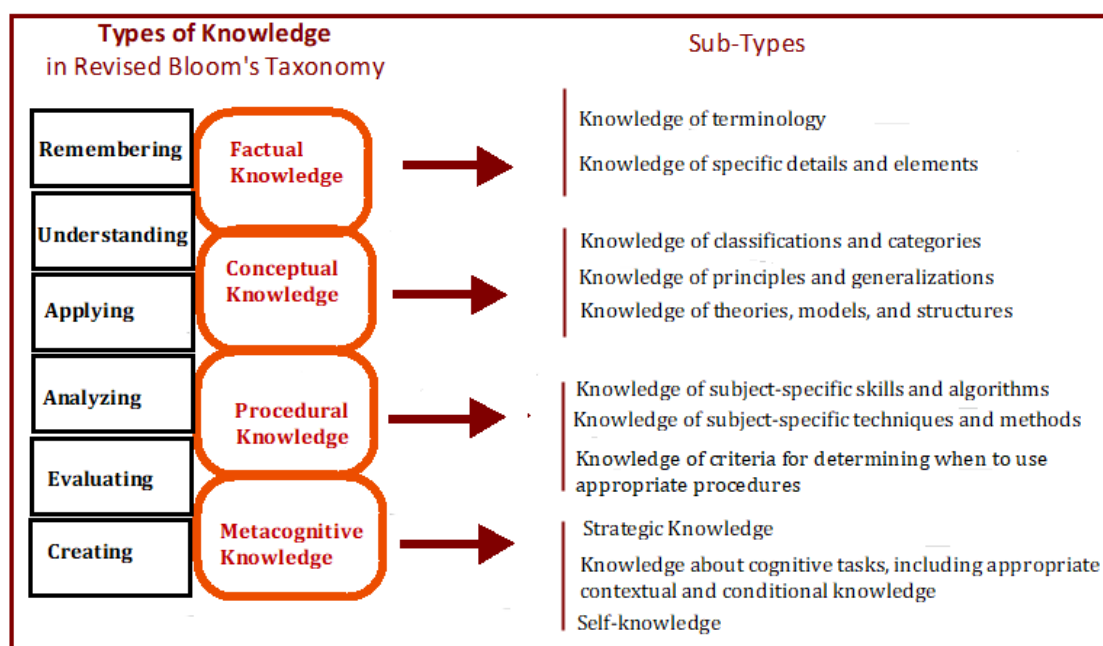
Bloom's taxonomy, established in 1956, serves as a framework for categorizing educational learning objectives. This framework was introduced in the publication titled "*Taxonomy of Educational Objectives, The Classification of Educational Goals Handbook I: Cognitive Domain.*" (Bloom et al., 1956). Bloom's taxonomy delineates three distinct domains: the *cognitive domain*, which emphasizes the enhancement of students' intellectual abilities, including skills such as information recall, concept evaluation, and the application of knowledge in innovative contexts; the *affective domain*, which pertains to the cultivation of students' attitudes, values, and interests; and the *psychomotor domain*, which is primarily concerned with the processing of sensory information and physical movement. In the following discussion, I will concentrate mainly on the cognitive domain, as it is particularly relevant to the application within secondary and post-secondary education.

**Table I: Cognitive Domain in Bloom's revised taxonomy (adapted from Anderson & Krathwohl, 2001)**

Cognitive Operation	Required actions articulated through the use of verbs	Cognitive Level
Remembering (Knowledge)	Retrieving, recognizing, and recalling relevant knowledge from long-term memory	Lower -order thinking
Understanding (Comprehension)	Constructing meaning from oral, written, and graphic messages through interpreting, exemplifying, classifying, summarizing, inferring, comparing, and explaining	
Applying (Application)	Carrying out or using a procedure through executing, or implementing.	
Analyzing (Analysis)	Breaking material into constituent parts, determining how the parts relate to one another and to an overall structure or purpose through differentiating, organizing, and attributing.	Higher -order thinking
Evaluating (Evaluation)	Making judgments based on criteria and standards through checking and critiquing.	
Creating (Synthesis)	Putting elements together to form a coherent or functional whole; reorganizing elements into a new pattern or structure through generating, planning, or producing	

The taxonomy proposed by Anderson & Krathwohl (2001) is an updated version that builds upon the foundational work of Bloom et al. (1956). The revised taxonomy modified categories

and incorporates action verbs linked to each of the six cognitive dimensions. The accompanying table illustrates the six facets of learning: *Remembering*, *Understanding*, *Applying*, *Analyzing*, *Evaluating*, and *Creating*, along with a concise description of the processes involved and verbs that educators can employ to encourage students to engage at these cognitive levels. This revised taxonomy encompasses two primary dimensions. The first dimension is known as the knowledge domain, which categorizes knowledge into four distinct types (Anderson & Krathwohl, 2001): *factual*, *conceptual*, *procedural*, and *metacognitive*. *Factual knowledge* refers to the essential components that students need to understand in order to engage with a particular discipline or address problems within it. *Conceptual knowledge* pertains to the connections among these fundamental elements, which allow them to operate cohesively within a broader framework. *Procedural knowledge* encompasses the processes involved in performing tasks, including methods of investigation and the standards for applying skills, algorithms, techniques, and procedures. *Metacognitive knowledge* is characterized by an understanding of cognition in a general sense, as well as an awareness and comprehension of one’s own cognitive processes.



**Figure 2.** Types and subtypes of knowledge described in the revised Bloom’s Taxonomy (adapted from Anderson & Krathwohl, 2001)

Marzano and Kendall (2007, Preface, xi) articulate that Bloom’s Taxonomy serves several distinct purposes: (1) it provides a framework for the design and classification of educational objectives, (2) it offers a structure for the designing of assessments, (3) it acts as a resource for enhancing the utility of state standards for educators, (4) it serves as a structure for curriculum design, and (5) it underpins a curriculum focused on thinking skills. This article seeks to equip teachers, educators, and researchers with a comprehensive understanding of the integration of AI tools (e.g., ChatGPT) and Bloom’s Taxonomy, emphasizing their effectiveness, aiming to enhance teaching approaches with innovative methods (e.g. brainstorming sessions, the design of formative assessments).

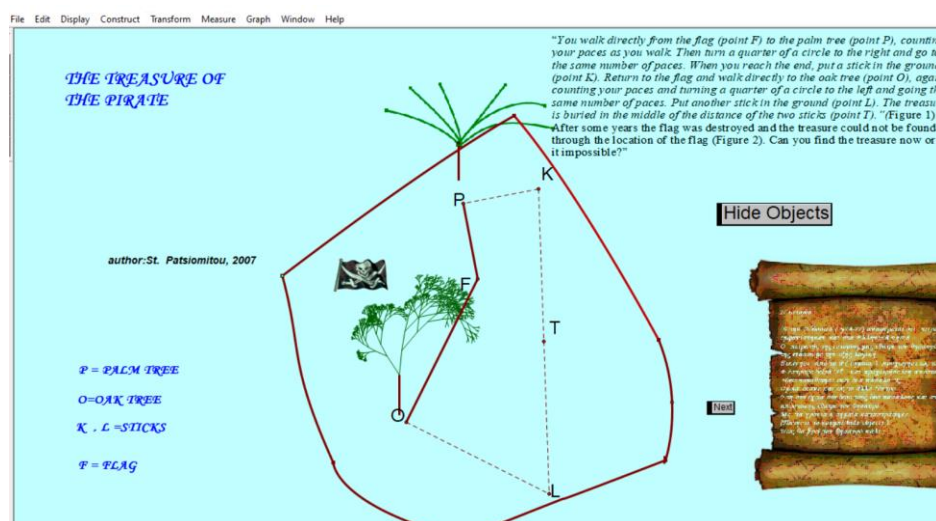


### 3.0 A DIGITAL SCENARIO THROUGH PROMPTS IN ARTIFICIAL INTELLIGENCE TOOLS

The objectives of the educational experiment described in the current digital scenario can be encapsulated in four key points: (a) To improve comprehension of the changing dynamics associated with the integration of AI in the didactics of mathematics. (b) To investigate the potential of AI in advancing sensemaking and fostering the development of mathematical reasoning. (c) To further examine the collaborative efforts involved in incorporating AI into classroom settings. (d) To analyze the impact of AI on mathematical reasoning and the creativity of students.

In the current study I employed the Problem A below, that I investigated in my earlier studies involving secondary class students (e.g., Patsiomitou, 2008a, b), giving to them the following assignment: to construct their figures utilizing either dynamic or static methods, which include the use of a compass and ruler or dynamic geometry software.

**Problem A:** The problem at hand is a reworking of the scenario originally presented by George Gamow, which features pirates and hidden treasure. Gamow's (1988) scenario revolves around a treasure map discovered in the attic of an elderly man. Below is the revised version (Patsiomitou, 2008a): *"In the Odyssey, Homer (c74-77) mentions that the pirates also raided Greek islands. The pirate in our story has buried his treasure on the Greek island of Thasos and noted its location on an old parchment: "You Walk directly from the flag (point F) to the palm tree (point P), counting your paces as you walk. Then turn a quarter of a circle to the right and go to the same number of paces. When you reach the end, put a stick in the ground (point K). Return to the flag and walk directly to the oak tree (point O), again counting your paces and turning a quarter of a circle to the left and going the same number of paces. Put another stick in the ground (point L). The treasure is buried in the middle of the distance of the two sticks (point T)." After some years the flag was destroyed and the treasure could not be found through the location of the flag. Can you find the treasure now or is it impossible?"*



**Figure 3.** The design of the problem in the DGS environment (e.g., Patsiomitou, 2008a, b, 2010)

The way in which I design the mathematical problems within the DGS environment has enabled me to coin, introduce and develop the dynamic concept of *Linking Visual Active Representations* (Patsiomitou, 2008a, b). I have reported this idea in many articles (e.g., Patsiomitou, 2008a, b, 2010, 2019 a, b, c, 2021a, b, 2023).

This study employs suitable “prompts” to theoretically sample data for AI systems related to education. The term "Prompt" is new in this context and refers to the creation of suitable questions for AI systems, such as ChatGPT, Leonardo.AI, Lumen5.AI, Pictory.AI etc. (websites [9, 10, 11, 12]). Many research centers (e.g., Coursera) have developed a catalog of prompting strategies for effectively managing ChatGPT (website [6]). In this study the methodology for utilizing prompts encompassed the subsequent steps:

I presented the issue (Problem A) to ChatGPT as a prompt and asked for the creation of a quiz designed for secondary school students. Subsequently, I introduced Problem A to Leonardo.AI, Pictory.AI, and Lumen5.AI, aiming to generate relevant images and videos. After receiving feedback, I returned to ChatGPT to request the development of a new issue (Problem B), incorporating the newly introduced elements. I then presented Problem B to Leonardo.AI, Pictory.AI, and Lumen5.AI for the production of updated images and videos. Additionally, I submitted Problem B as a prompt in ChatGPT, requesting the creation of a dialogue involving secondary or tertiary students. Following this, I sought the formulation of a dialogue that included secondary or tertiary students and a mathematics teacher. I subsequently analyzed the quiz and dialogues generated by ChatGPT through the framework of Bloom's revised taxonomy. In conclusion, I reported my findings.

Presented below is an excerpt from the “conversation” with ChatGPT. The prompts I provide to ChatGPT, along with the responses from ChatGPT, are formatted in italics. In the discussion, my name is represented with capitalized initial letters ([S.P.]).

**S.P. prompt:** *A video showcases a mathematical challenge known as the treasure hunting problem, in which a pirate has misplaced his treasure and is now employing his mathematical abilities to locate it. Additionally, the video features the use of a tablet and an electronic navigator. Could you generate a modern version of the treasure-hunting math problem?*

**ChatGPT said:** *Thank you for the description! Based on this, I'll create a modern version of the treasure-hunting math problem with a tech-savvy twist. Here's how the problem could go.*

**Problem B** (generated by ChatGPT, based on Problem A): *Captain Ruby, a renowned modern-day treasure hunter, has discovered an old pirate map on her tablet while navigating the Caribbean Sea using her state-of-the-art electronic navigator. The map shows the location of a lost treasure buried on an uncharted island. The treasure is hidden at a specific location that can be determined using the following clues: **Clue 1:** The treasure is buried 200 meters north of the ancient palm tree, which is located at the coordinates (35°N, 120°W). **Clue 2:** From the palm tree, the treasure is 300 meters east, but the path is along a curved shoreline that Captain Ruby can measure using her electronic navigator. **Clue 3:** The island has a magnetic anomaly, and the treasure lies on the perpendicular bisector of the line segment connecting two rocks, which are located at (35.0002°N, 120.0003°W) and (35.0005°N, 120.0001°W). **Tasks: Calculate the Treasure's Coordinates:** First, determine the coordinates after moving 200 meters north and then 300 meters east from the ancient palm tree using the appropriate trigonometric functions for distance on the Earth's surface. Then, find the perpendicular*

bisector of the line segment connecting the two rocks. Use this to adjust the final location, considering the effect of the magnetic anomaly. **Determine the Distance:** Calculate the distance from the palm tree to the treasure's final location along the curved shoreline. **Analyze the Impact of the Magnetic Anomaly:** Determine how much the magnetic anomaly shifts the final location of the treasure by comparing the actual magnetic north to true north. **Solution Outline:** Use the Haversine formula or another appropriate method to find the treasure's approximate latitude and longitude after moving north and east. Determine the perpendicular bisector's equation and adjust the coordinates. Compute the total distance traveled using the curved path.

The aforementioned Problem B generated by ChatGPT integrates modern technology like tablets and electronic navigation, giving it a contemporary feel while maintaining the classic adventure of treasure hunting.

The integration of AI tools is expected to transform the study's approach. The research questions that will be investigated are the following:

- *Do artificial intelligence tools assist students in visualizing problems? What are the advantages and disadvantages of these AI-based image generation tools?*
- *Can these AI tools potentially enhance and motivate practices related to the problem-solving process and the development of students' reasoning abilities?*
- *Does the incorporation of artificial intelligence in education signify a major shift in teaching methodologies?*

Therefore, in the present research, I aim to explore the potential of AI tools that may assist in the problem-solving process. Additionally, I will examine, which aspects of the problem, AI identifies as important and how these aspects can be incorporated into the quiz generation process facilitated by ChatGPT. In addition, what contemporary and inventive features might ChatGPT integrate into a redefinition of the problem (such as utilizing a tablet or an electronic device for the treasure hunt). A crucial aspect is the dialogue that ChatGPT could generate between two students, and subsequently between these students and a mathematics professor.

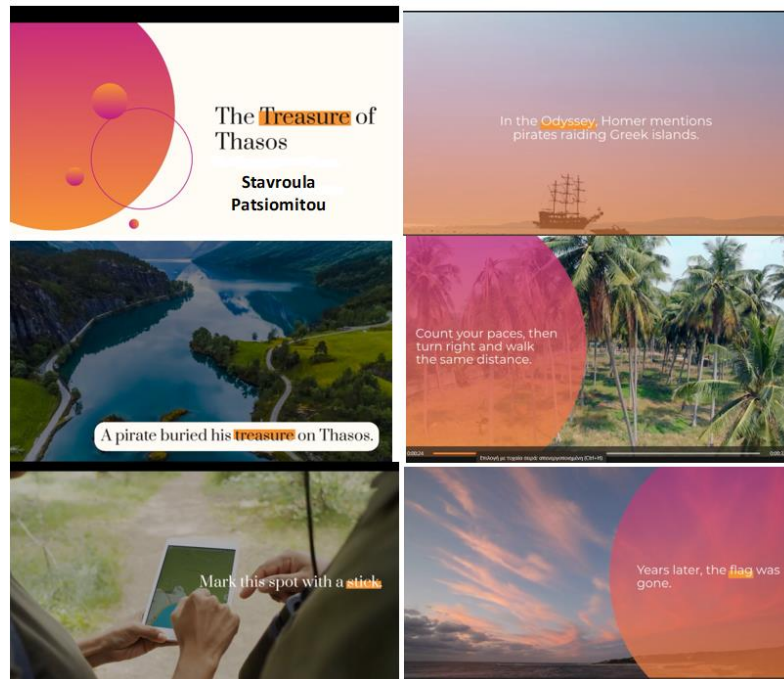
Subsequently, the process of theoretical sampling involved the following steps (Patsiomitou, 2024): (a) formulating appropriate prompts/questions to elicit responses from ChatGPT, images from Leonardo.AI and videos from Lumen5.AI or Pictory.AI; (b) reviewing the questions based on the data from ChatGPT's responses and adding new prompts as needed; (c) verifying the accuracy of the responses through appropriate online investigations; (d) checking for potential plagiarism of the responses from published work; and (e) compiling the responses and drawing conclusions.

A summary review of the study and its findings follows, along with the development of a "theory" on the use of artificial intelligence as a tool for professional development of educators and improving the quality of educational work. According to Glaser & Strauss (1967), *"theoretical sampling is a process of data collection for generating theory, where the analyst jointly collects, codes, and analyzes data and decides on the type of data to be collected subsequently to develop the theory."*

The educational experiment will facilitate the derivation of conclusions regarding how ChatGPT utilizes Bloom's cognitive domain in formulating questions and fostering subsequent discussions among students and educators, stemming from a series of increasingly complex







**Figure 5.** Using Lumen.5 for the generation of a video

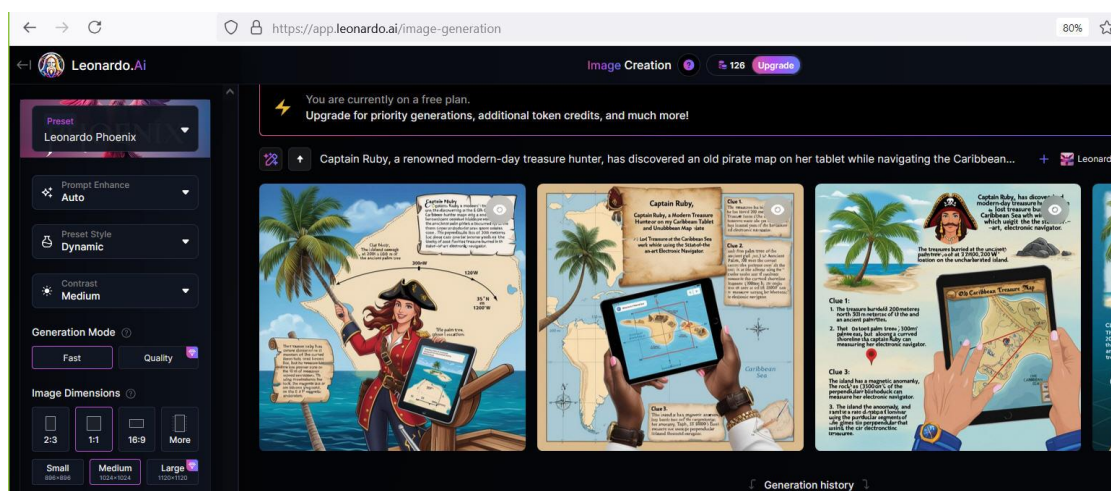
According to Shen-yi Liao & Tamar Gendler (2020), the act of imagining involves creating representations that do not necessarily reflect the current, objective, or subjective reality. Consequently, imagination can be employed to depict alternatives to what is real, to envision moments beyond the present, and to convey viewpoints that differ from one's own. Furthermore, the incorporation of real-world problems serves as a cognitive tool in mathematics education, facilitating students' connections between abstract ideas, algorithms, and procedures. This approach not only enhances their understanding but also renders the learning experience more meaningful. It encompasses the capability to transcend the physical and to conceive both what is possible and what may appear to be out of reach. This cognitive ability is defined by the interaction between actual circumstances and possibilities, an interplay between reality and potentiality the known and the novel, as well as the logical and the illogical.



**Figure 6.** Using Pictory.AI for the generation of a video

The significance of experiential learning in fostering creativity and innovation is particularly evident in the realm of mathematics education. Creativity cannot be developed solely through theoretical understanding; it necessitates the involvement of students in activities that promote creative expression, risk-taking, and the exploration of non-traditional methods. While imagination is often associated with the arts, it is equally essential in mathematics, as it enables students to comprehend concepts from various viewpoints. According to Peirce's (1906) theoretical viewpoint, images or diagrams serve as representations of cognitive visualization and mental imagery that are subject to mental transformations and reinterpretations, functioning as instruments for expressing the meanings attributed to them by the individual. Peirce (1906) asserts that a diagram serves as an icon representing a collection of rationally interconnected objects, which not only depicts the related elements but also, and more significantly, illustrates the relationships among them. Diezmann (2000, 2005), on the other hand, posits that images /diagrams offer three significant cognitive benefits in the realm of problem solving: they aid in conceptualizing the structure of the problem, an essential phase in achieving a successful resolution; they function as a knowledge representation system that enables inference-making and knowledge generation; and they enhance visual reasoning, which, while complementary to linguistic reasoning, remains distinct from it.

The function of diagrams in justification differs according to the learner's level. Students at lower levels tend to view diagrams holistically, using them to formulate arguments and create informal proofs based on the elements present in the diagram (e.g., van Hiele, 1986). In contrast, students at higher levels possess a deeper understanding of a diagram's properties and recognize that it symbolizes abstract geometric ideas. Barwise & Etchemendy (1991) examined the significance of images or diagrams in the proof and proving process, concluding that diagrams, similar to sentences, convey information by partitioning the same set of capabilities expressed in various, distinct ways. Thus, they can be essential in reasoning and serve as valid components in mathematical problem solving. To engage in geometric proofs, students must go beyond mere visual observation and connect diagrams to established geometric concepts. Therefore, it is essential to assist students in enhancing their skills in interpreting diagrams. Consequently, the objective is to create engaging scenarios that enhance reasoning and argumentation abilities.



**Figure 7.** Using Leonardo.AI for the generation of images related to Problem B



In my opinion, to assist students in addressing misconceptions arising from AI-generated images, visual elements, or word-based data, educators can employ a range of strategies.

- One effective approach is to facilitate a thorough analysis of the AI-generated image. Instruct students to pinpoint any inaccuracies or inconsistencies related to the mathematical concepts depicted. Encourage them to consider questions such as, "What observations can you make regarding the arrangement of the objects?" and "Is the rotation accurate?" This process enhances students' critical thinking skills.
- Another approach enhances the discussion among students in their small groups. Arrange students into small groups to discuss and analyze the errors they observe and generate alternative representations. Encourage them to juxtapose the AI-generated image with their own comprehension of the issue, thereby promoting collaborative learning and the sharing of ideas.
- Assist students in articulating the problem through various forms of representation, including diagrams, physical models, and verbal explanations. This approach enables students to view the problem from different viewpoints and gain a deeper understanding of the fundamental concepts, incorporating hands-on activities. In essence, this approach involves employing multiple representations.
- Provide guidance on effective visualization techniques, emphasizing the accurate representation of rotations, proportions, and spatial relationships. Discuss frequent mistakes in visual representation and explain common pitfalls in visual representation and how to avoid them.
- Once students have engaged in discussion and attempted to rectify the inaccuracies, deliver corrective feedback by presenting a precise representation of the problem. Clarify the correct positioning of elements and the accurate depiction of rotations, thereby reinforcing their understanding.
- Encourage students to contemplate their thought processes and reasoning when identifying mistakes and suggesting solutions. This practice can enhance their awareness of personal misconceptions, foster the development of robust problem-solving abilities, and promote metacognitive reflection.

Through the implementation of these strategies, the teacher can assist students in cultivating a more profound comprehension of mathematical concepts, thereby enhancing their capacity to visualize and accurately resolve problems with the aid of AI tools.

## **5.0 USING CHATGPT FOR THE GENERATION OF QUIZZES**

At the conclusion of the educational process, educators aspire for every student to have achieved proficiency in the relevant concepts and skills, enabling them to effectively utilize this knowledge in various problem-solving contexts. Summative evaluation is an assessment that follows instruction that provides information about mastery of learning targets. Traditionally quizzes were used for summative evaluation. A crucial component is self-assessment, which plays a vital role in facilitating effective learning. This process empowers students to take charge of their educational journey and cultivate the skills required for autonomous learning. Self-assessment takes place when students reflect on and assess their own work. Research has also investigated the efficacy of ChatGPT in evaluation processes. In this study, the approach involved administering quizzes to mathematics classes. The effectiveness of ChatGPT in assessments has also been examined in studies. The intervention

used in this study was to assign quizzes to mathematics classes. The examination of the quiz questions will align with the descriptions provided by NCTM (2014) (Table II, adapted), while the evaluation of the anticipated responses will be framed through Bloom’s cognitive taxonomy.

**Table II:** Types of questions, their explanations, and the classification of knowledge

Type of question	Descriptions- explanations	Knowledge classification
Recalling information	Students recall information, definitions, or processes.	Factual knowledge
Investigating and probing thought processes	Students articulate, expand upon, or elucidate their thought processes, which includes detailing the procedures involved in solving problems or completing assignments.	Conceptual knowledge
Establishing mathematical interrelationships	Students engage in discussions regarding mathematical structures and establish connections between various mathematical concepts and their interrelationships.	Conceptual and procedural knowledge
Encouraging reflection and logical thinking	Students demonstrate a more profound comprehension of their reasoning and actions, which includes justifying the validity of their work.	Metacognitive knowledge

**Table III:** A ChatGPT quiz accompanied with a brief evaluation utilizing Bloom’s revised taxonomy.

S.P. prompt: <i>Would you be able to create a quiz pertaining to the aforementioned problem? This problem is addressed to secondary school students.</i>	
ChatGPT quiz	Brief evaluation by S.P.
(The remarks I included elucidate several theoretical issues)	
<p><b>1. What is the main challenge in finding the treasure on the island of Thasos?</b></p> <p>a) The palm tree has been destroyed                      b) The flag (point F) has been destroyed                      c) The oak tree has been destroyed                      d) The old parchment is lost</p> <p>This question tests students’ recall of specific information from the problem context.</p>	<p><i>Remembering Knowledge of specific details and elements</i></p> <p>.....</p> <p><i>Factual knowledge</i></p>
<p><b>2. According to the pirate's instructions, how do you determine the location of the first stick (point K)?</b></p> <p>a) Walk from the flag to the oak tree, turn right, and walk the same number of paces                      b) Walk from the palm tree to the flag, turn left, and walk the same number of paces                      c) Walk from the flag to the palm tree, turn right, and walk the same number of paces                      d) Walk from the flag to the oak tree, turn left, and walk the same number of paces.</p> <p>This question assesses understanding of the instructions and the relationship between points and directions.</p>	<p><i>Recalling information, Knowledge of principles and generalizations</i></p> <p>.....</p> <p><i>Conceptual knowledge (Investigating thought processes)</i></p>



<p><b>3.How is the location of the treasure (point T) determined?</b>                  a) By finding the midpoint between the oak tree and the palm tree                  b) By finding the midpoint between the flag and the oak tree                  c) By finding the midpoint between points K and L                  d) By walking halfway between the flag and the palm tree.</p>	<p><i>Understanding                  Hypothesising                  -Establishing mathematical interrelationships                  .....</i></p>
<p>This question tests students' ability to apply <b>procedures</b> related to finding midpoints or following a set of instructions to locate a specific point.</p>	<p><i>Procedural knowledge                  Knowledge of subject-specific skills and algorithms</i></p>
<p><b>4. If the flag (point F) is destroyed, what should be the initial approach to finding the treasure?</b>                  a) Estimate the original location of the flag based on memory                  b) Give up on finding the treasure, as it is impossible without the flag                  c) Use the distances between the trees to recreate the flag's location                  d) Measure the distance between the trees and find a new flag location</p>	<p><i>Critiquing                  Applying                  Encouraging reflection and reasoning                  .....</i></p>
<p>This question evaluates the student's understanding of the steps or procedures that should be followed if a key element is missing.</p>	<p><i>Procedural knowledge                  Knowledge of subject-specific skills and algorithms</i></p>
<p><b>5.What is the geometric relationship between the points K, L, and T?</b>                  a) K and L are equidistant from the flag                  b) T is the centroid of triangle formed by K, L, and F                  c) T is the midpoint of the line segment connecting K and L                  d) K and L are on the same line as the flag</p>	<p><i>Analysing                  Hypothesizing                  Knowledge of principles and generalizations                  .....</i></p>
<p>This question requires students to understand and identify the geometric relationships between specific points.</p>	<p><i>Conceptual knowledge                  Encouraging reflection and reasoning</i></p>
<p><b>6.Is it possible to find the treasure if the flag is destroyed?</b>                  a) Yes, because the locations of the trees can help reconstruct the flag's location                  b) No, because the exact location of the flag is essential                  c) Yes, but only if you remember the distance between the flag and the trees                  d) No, unless you have another way to locate point T directly.</p>	<p><i>Analysing                  Critiquing                  Planning                  Encouraging reflection and reasoning                  .....</i></p>
<p>This question involves recalling specific information from memory, such as the destruction of a key landmark.</p>	<p><i>Metacognitive knowledge -                  Strategic knowledge</i></p>
<p><i>Short Answer Questions:                  Describe the step-by-step process to find the location of the treasure as per the pirate's instructions.</i></p>	<p><i>Factual knowledge</i></p>
<p><i>Explain why the destruction of the flag complicates the search for the treasure.</i></p>	<p><i>Conceptual knowledge</i></p>
<p><i>Given the trees are still standing, how might one use their positions to approximate the flag's original location?</i></p>	<p><i>Procedural knowledge</i></p>
<p><i>What mathematical concepts can be applied to solve this problem geometrically if the flag is missing?</i></p>	<p><i>Conceptual knowledge</i></p>
<p><i>Is there a way to solve this problem without knowing the original position of the flag? Justify your answer.</i></p>	<p><i>Metacognitive knowledge</i></p>

The six cognitive processes in revised Bloom’s Taxonomy involved in the quiz are: (a) Remembering (Questions at this level are straightforward and test memory). (b) Understanding (Involves explaining ideas or concepts in one's own words). (c) Applying (Using knowledge in a new but familiar situation). (d) Analyzing (Breaking down complex information into

components to understand its structure). (e) Evaluating (Making judgments based on criteria or standards). (f) Creating (Generating new ideas, products, or ways of understanding).

### 6.0 GENERATING A MATHEMATICAL DISCUSSION USING CHATGPT

The integration of ChatGPT in educational contexts presents challenges, particularly regarding the dissemination of inaccurate information and the potential for academic dishonesty. Additionally, excessive dependence on ChatGPT may hinder students' active participation in their learning processes, as it could limit their opportunities for critical thinking and problem-solving. Consequently, it is crucial to examine the impact of ChatGPT on student engagement to enhance its application in educational environments and inform future research initiatives related to ChatGPT-assisted learning. The subsequent section of the educational experiment will showcase an excerpt of a hypothetical mathematical dialogue between two university students, Alex and Mia (names assigned by ChatGPT). Following this exchange, a professor will also join the discussion. The prompt utilized, along with remarks/or comments pertaining to the scientific concepts addressed in the conversation, are reported in the table below. Additionally, a concise analysis employing Bloom’s cognitive domain is included. Specifically, a classification of the dialogue among the students is presented based on the six cognitive processes outlined in Bloom's taxonomy.

Table IV: Dialogues produced by ChatGPT along with a concise analysis	
<p><b>S.P. prompt:</b> <i>Two students are engaged in reading a story from a journal, which prompts them to initiate a discussion about mathematics. Please envision and articulate the conversation they might have with one another.</i></p> <p><b>ChatGPT:</b> <i>Sure! Here’s how the conversation between Alex and Mia, might unfold as they discuss the math problem from the treasure-hunting story.</i></p>	
Students dialogue (The remarks I included elucidate several theoretical issues)	Brief analysis by S.P.
<p><b>Alex:</b> <i>"Wow, this treasure hunt story is so cool! I love how Captain Ruby <b>uses her tablet and electronic navigator</b> to find the treasure. But the math part... it's kinda tricky, don't you think?"</i></p> <p><b>Mia:</b> <i>"Yeah, but I think we can figure it out. The first thing Captain Ruby does is move 200 meters north from the palm tree. We know that on Earth, moving north or south <b>changes the latitude, but not the longitude</b>".</i></p> <p>Comments: Geographic coordinates, specifically longitude and latitude, serve as essential tools for accurately identifying locations on the surface of the Earth.</p>	<p><i>Factual knowledge (Knowledge of specific details and elements)</i>  <i>Conceptual Knowledge (Knowledge of principles and generalizations)</i></p>
<p><b>Alex:</b> <i>"Right! So if the palm tree is at (35°N, 120°W), moving 200 meters north will just <b>increase the latitude a bit</b>. But how do we calculate that exactly?"</i></p> <p><i>(Alex understands that movement north affects latitude, not longitude, and can explain how this affects their position- <b>Understanding</b>)</i></p>	<p><i>Factual knowledge</i>  <i>Conceptual Knowledge <b>and</b> Procedural Knowledge (Knowledge of subject-specific skills and algorithms)</i></p>

<p><b>Mia:</b> "We could use the Earth's circumference to figure out how much one degree of latitude is in meters. I remember reading that <b>one degree of latitude is about 111 kilometers</b>. So, 200 meters would be just a tiny fraction of a degree."</p> <p>(Mia is recalling a specific fact about latitude-<b>Remembering</b>).</p> <p>Comments: Should the Earth be considered a flawless sphere, one degree of latitude, called an arc degree, covers about 111 kilometers (69 miles). ( <a href="https://education.nationalgeographic.org/resource/latitude/">https://education.nationalgeographic.org/resource/latitude/</a>)</p>	
<p><b>Alex:</b> "Oh, that makes sense! So, we'd just add that fraction to the 35°N. What about moving 300 meters east? I know <b>longitude changes when you move east or west</b>, but it's more complicated because the Earth is round."</p> <p>(Alex applies the concept of latitude change to their treasure hunt scenario-<b>Applying</b>).</p> <p><b>Mia:</b> "Exactly. <b>Longitude lines get closer together as you move towards the poles.</b> "(Knowledge of principles and generalizations). We could use <b>trigonometry or maybe the Haversine formula</b> to find out how much the longitude changes. But remember, the navigator said the path was along a curved shoreline. That might make it more complex.</p> <p>(Mia applies mathematical formulas, trigonometry and the Haversine formula to a real-world situation- <b>Applying</b>)</p> <p>Comments: The haversine formula calculates the great-circle distance between two points on a sphere based on their longitudes and latitudes. This formula is significant in navigation and represents a specific instance of a broader formula in spherical trigonometry known as the law of haversines, which connects the sides and angles of spherical triangles. (<a href="https://en.wikipedia.org/wiki/Haversine_formula">https://en.wikipedia.org/wiki/Haversine_formula</a>)</p>	<p>Conceptual knowledge</p> <p>Application of Technology in Problem-Solving</p> <p>Knowledge of theories, models, and structures</p>
<p><b>Alex:</b> "True. So, the navigator helps measure the curve accurately. But there's also the <b>perpendicular bisector clue</b>. We need to find the <b>midpoint between the two rocks</b> and then the perpendicular bisector of that line. Do you remember how to do that?"</p> <p><b>Mia:</b> "I think so! The midpoint is easy; it's just the average of the coordinates of the rocks. But finding the perpendicular bisector... we'd need the slope of the line connecting the rocks, and then the slope of the perpendicular bisector would be the negative reciprocal of that." [...]</p> <p>Comments: A negative reciprocal refers to the inverse of a number, accompanied by a change in sign. For instance, the negative reciprocal of 4/5 is -5/4.</p>	<p>Conceptual knowledge</p> <p>Contextual Understanding (Knowledge of criteria for determining when to use appropriate procedures) <b>Remembering- Applying -Cognitive Process</b></p>
<p>This dialogue shows how Alex and Mia discuss the math concepts involved in the treasure-hunting problem while making connections to real-world applications. It also demonstrates their curiosity and teamwork as they engage with the problem. Overall, I noted that it is possible to develop a summary based on the revised Bloom's taxonomy: (a) <i>Remembering</i>: Mia demonstrates her recall of <i>factual information</i> regarding latitude and the Earth's circumference. (b) <i>Understanding</i>: Alex and Mia exhibit <i>comprehension</i> by articulating the influence of latitude and longitude on movement. (c) <i>Applying</i>: Both students <i>utilize</i> their knowledge of geometry, trigonometry, and navigation to address the problem at hand. (d) <i>Analyzing</i>: Mia and Alex <i>examine the interrelationships</i> among points, slopes, and how the Earth's shape impacts movement. (e) <i>Evaluating</i>: Alex <i>assesses</i> the effect of the magnetic anomaly on their calculations. (f) <i>Creating</i>: Mia formulates a new solution strategy <i>employing mathematical principles</i> to determine the coordinates of the treasure.</p>	

<p><b>S.P. prompt:</b> A mathematics Professor is now participating in their discussion, thereby elevating its formality. What inquiries might the teacher pose to the aforementioned students?</p> <p><b>ChatGPT said:</b> Certainly! Here's how the conversation might evolve when their math teacher, Ms. Carter, joins in.</p>	
<p><b>The discussion generated by ChatGPT</b> (The comments /remarks I included elucidate several theoretical issues)</p>	<p><b>Brief analysis by S.P.</b></p>
<p><b>Ms. Carter:</b> "Hello, Alex, Mia! I overheard your discussion about the treasure hunt problem—you're doing an excellent job of breaking it down. I see you're both thinking critically about the math involved. Let's dig a little deeper into some of these concepts, shall we?"</p> <p><b>Alex:</b> "Sure, Ms. Carter! We were just trying to figure out how Captain Ruby could use the coordinates and the clues to find the treasure."</p>	
<p><b>Ms. Carter:</b> "Great! So, you've already discussed moving north 200 meters from the palm tree. Alex, can you explain how you would calculate the change in latitude using what you know about the Earth's circumference?"</p> <p>Comments: The circumference of the Earth refers to the total distance encircling the planet. When measured at the equator, this distance is approximately 40,075.017 kilometers. <a href="https://en.wikipedia.org/wiki/Earth%27s_circumference">https://en.wikipedia.org/wiki/Earth%27s_circumference</a></p> <p>.....</p>	<p>Factual knowledge</p> <p><b>Knowledge of specific details and elements</b></p>
<p><b>Alex:</b> "Sure! Since one degree of latitude is about 111 kilometers, I figured 200 meters would be a small fraction of a degree."</p> <p><b>Ms. Carter:</b> "Exactly right. The Earth's circumference at the equator is about 40,075 kilometers, so your calculation is spot on. Now, Mia, how would you handle the change in longitude after moving 300 meters east? What makes this calculation more complex?" <b>Mia:</b> "Well, since longitude lines get closer together near the poles, the distance corresponding to a degree of longitude depends on the latitude. We could use the <b>cosine of the latitude</b> to adjust for this. So, first, we'd calculate how much <b>one degree of longitude is in meters</b> at that latitude, and then find the fraction for 300 meters." [...]</p> <p>.....</p>	<p>Conceptual and procedural knowledge- <b>(Knowledge of principles and generalizations [...])</b></p> <p>.....</p> <p>Exchange Ideas Effectively - <b>(Knowledge of subject-specific skills and algorithms)</b></p> <p>.....</p> <p>Construct Knowledge Collaboratively <b>(Knowledge of principles and generalizations).</b></p>
<p><b>Alex:</b> "Wow, I didn't realize how much math was involved in something like this. It's pretty awesome when you think about it!"</p> <p><b>Ms. Carter:</b> "It is! And this is why learning these concepts is so important—they're not just abstract ideas, but tools you can use in adventures, problem-solving, and even future careers. Great job, both of you. Keep thinking critically, and math will always be an adventure!"</p>	<p>Metacognitive Knowledge-<b>(Self-knowledge)</b></p>

### 7.0 CHATGPT AND THEORETICAL PERSPECTIVES

Bloom's Taxonomy serves as a structured framework that classifies various levels of cognitive skills, ranging from fundamental knowledge recall to complex, intricate analysis and evaluation. In the field of education, it is employed to create assessments, lessons, and activities aimed at engaging different cognitive processes. When generating quizzes or questions, a

system such as ChatGPT can utilize this framework to guarantee that the questions address a spectrum of cognitive abilities, thereby promoting deeper /enhanced understanding and critical thinking. I present several conclusions regarding the potential relationships between various cognitive processes and types of knowledge in relation to ChatGPT quiz questions.

- **Factual Knowledge:** Questions at this level assess the recall of fundamental facts, terminology, and specific details, focusing on the retention of information.
- **Conceptual Knowledge:** This category pertains to the comprehension of concepts, categories, theories, and models, requiring students to grasp and articulate these ideas.
- **Procedural Knowledge:** Questions in this domain evaluate the capability to implement specific methods, techniques, or algorithms to address problems, thereby assessing the application of these approaches.
- **Metacognitive Knowledge:** This level encompasses an awareness of one’s cognitive processes and learning strategies, prompting reflection on problem-solving techniques or learning methodologies and strategies.

It was essential for me to integrate the responses from ChatGPT concerning its application of various theoretical frameworks. At this point, I posed questions to ChatGPT using the prompts outlined in Table V.

<b>Table V:</b> ChatGPT and theoretical perspectives
<p><b>S.P. prompt:</b> <i>I have come to understand that ChatGPT employs Bloom's taxonomy in the creation of quizzes. Is this accurate? Could you provide an explanation?</i></p> <p><b>ChatGPT replied:</b> <i>When designing quizzes, I can generate a variety of questions targeting different levels of cognition from Bloom's Taxonomy. For example:</i></p> <ul style="list-style-type: none"> <li>• <i>A quiz may start with a few <b>remembering</b> or <b>understanding</b> questions to test basic knowledge.</i></li> <li>• <i>Then, it might move into <b>application</b> or <b>analysis</b> to test whether students can use or dissect this knowledge.</i></li> <li>• <i>Finally, it could include <b>evaluative</b> or <b>creative</b> questions that require deeper critical thinking or synthesis of concepts.</i></li> </ul> <p><i>This systematic approach ensures a balanced assessment of different cognitive skills, from basic recall to higher-order thinking, which aligns with Bloom's framework.</i></p>
<p><b>S.P. prompt:</b> <i>According to Vygotsky's (1978) sociocultural theory, both psychological and cultural tools serve as crucial mediators in the process of cognitive development. What significance do these tools hold within the context of the discussion?</i></p> <p><b>ChatGPT replied:</b> <i>The tools incorporated in Alex and Mia's discussion play a pivotal role in facilitating their understanding, problem-solving abilities, and collaborative learning. Here's how the tools function within their conversation. The tools in Alex and Mia's discussion serve as essential mediators that:</i> (a) <i>Facilitate Cognitive Development: They enable the students to extend their thinking and problem-solving abilities.</i> (b) <i>Support Collaborative Learning: Shared tools provide a basis for effective communication and joint problem-solving.</i> (c) <i>Bridge Theory and Practice: Applying abstract mathematical tools to a practical scenario enhances understanding and retention.</i> (d) <i>Encourage Higher-Order Thinking: The use of complex tools promotes analysis, synthesis, and creativity.</i></p> <p><i>By incorporating these tools, Alex and Mia not only solve the immediate problem but also engage in a deeper learning process that aligns with Vygotsky's emphasis on the importance of cultural and psychological tools in cognitive development. The tools empower them to operate within and even expand their Zone of Proximal Development, fostering both individual and collaborative growth.</i></p>



With this answer I realized that ChatGPT uses more theoretical frames. At this point the discussion was too long. Among others ChatGPT asked me to select between two different answers/replies, explaining that “You’re giving feedback on a new version of ChatGPT”. I found both approaches valuable as they offered a unique lens for analyzing students’ discussion as (a) Bloom's Taxonomy helps break down the cognitive processes involved, focusing on the progression from basic recall to higher-order thinking like analysis, evaluation, and creation. It provides a way to understand the levels of complexity in Alex and Mia’s problem-solving. (b) Vygotsky's Sociocultural Theory shifts the focus to the social and interactive aspects of learning, emphasizing how the students build knowledge together, use tools, and operate within each other's Zone of Proximal Development (ZPD) (Vygotsky, 1978). It highlights the importance of collaboration and external aids (tools) in learning.

## 8.0 CONCLUSIONS

It was essential for me to obtain ChatGPT's insights concerning the concept of Linking Visual Active Representations (e.g., Patsiomitou, 2008a, b, 2010, 2019 a, b, c, 2021 a, b, 2023).

**S.P. prompt:** *I have coined the idea of Linking Visual Active Representations. What are your thoughts? Do various tools such as Lumen5.AI, ChatGPT, and Leonardo.AI or Pictory.AI facilitate the creation of LVAR?*

**ChatGPT replied:** *The concept of **Linking Visual Active Representations (LVAR)** is a fascinating one, as it emphasizes the integration of visual, dynamic, and interactive elements to enhance learning and understanding. LVARs typically combine visual stimuli (like diagrams, animations, or graphics) with active engagement (e.g., manipulation, interaction, or exploration) to help learners build deeper, more meaningful representations of complex ideas. The tools you mentioned—Lumen5, ChatGPT, and Leonardo.AI—all contribute to the creation and facilitation of Linking Visual Active Representations (LVAR) in different ways. (a) Lumen5. AI or Pictory. AI: Promotes visual and narrative thinking, helping students conceptualize ideas through storytelling and animation. (b) ChatGPT: Encourages critical dialogue, problem-solving, and reflection, supporting both conceptual clarity and metacognitive growth. (g) Leonardo.AI: Enhances visualization, creative experimentation, and spatial reasoning by translating abstract ideas into visuals. Together, these tools help students develop more **robust mental models** and improve their capacity to reason, solve problems, and engage creatively with complex ideas.*

I emphasized the need to submit more intricate prompts to ChatGPT. For instance:

**S.P. prompt:** *In what ways does Lumen5 assist LVAR, or how does ChatGPT facilitate LVAR, or how does Leonardo.AI contribute to LVAR?*

I present a summary of the extensive responses I obtained from ChatGPT, highlighting the most significant points:

- Lumen5 generates visual representations from textual content by combining images, videos, and animations with written material. This approach aligns with the idea of LVAR, which allows learners to associate verbal /textual information with visual representations. For instance, a mathematical problem or a complicated concept can be converted into a brief animated video that visually demonstrates each step. The video

output from Lumen5 can simplify intricate information into visual components, thereby reinforcing the notion of active representations.

- ChatGPT primarily provides textual support, which can be effectively combined with LVAR through prompts that stimulate learners to visualize or construct mental models while interacting with the material. For instance, ChatGPT can encourage learners to visualize or create mental models while engaging with the content, therefore linking visual descriptions to active problem-solving: Although ChatGPT isn't inherently visual, it can prompt learners to actively engage with visual tools.
- On the other hand, Leonardo.AI produces static images that can transform into interactive elements/components when incorporated into dynamic educational platforms. By generating distinctive images based on specific textual inputs, Leonardo.AI can animate abstract concepts, enabling learners to perceive direct visual relationships between ideas, thereby fostering a deeper understanding through LVAR.

**S.P. prompt:** *What function do AI tools serve in the creation of LVAR? Are there any constraints, limitations or potentials associated with their use?*

**ChatGPT replied:** (i) **The role of AI tools in generating LVAR:** (a) **Scaffolding Understanding:** *AI tools like ChatGPT and Leonardo.AI can provide dynamic scaffolds that guide learners through complex concepts, linking visual elements to explanations.*

(b) **Engagement Through Interaction:** *While some of the tools like Lumen5 focus more on passive viewing, when integrated into interactive environments, they can shift from passive to active engagement, essential for LVAR.* (c) **Customization and Personalization:** *AI tools allow for the personalization of learning materials, which is a key aspect of LVAR. Learners can interact with representations that are directly relevant to their own learning context, making the representations more active.*

(ii) **Limitations and Potentials for Improvement:** (a) **Interactivity:** *While Lumen5, ChatGPT, and Leonardo.AI are excellent at generating content, they lack the built-in interactivity needed for full LVAR development.* (b) **Tool Integration:** *The tools could be more powerful if they are combined into a single learning platform where students can move between text, static images, and interactive video easily, or manipulate these elements dynamically (e.g., by dragging, rotating, and exploring visual elements).*

In my opinion, while each tool has its strengths, the real power of LVAR comes from combining these tools to create an integrated, learning experience that not only presents visual information but also encourages active engagement. These tools, especially when combined with interactivity, provide an evolving framework that aligns well with the goals of LVAR in promoting deeper understanding and dynamic learning experiences. Lumen5, ChatGPT, and Leonardo.AI etc. each play a significant role in developing students' thinking skills. They engage different cognitive processes such as critical, creative, analytical, conceptual, procedural, metacognitive, and integrative thinking. By offering multimodal learning opportunities—text-based interaction, visual creation, and multimedia synthesis—these tools enable students to approach problems and concepts from multiple viewpoints, enhancing their cognitive flexibility and depth of understanding.

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