

THE TREASURY OF ATREUS ‘RELIEVING’ TRIANGLE: INSIGHTS FROM HYPERBOLIC GEOMETRY

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ABSTRACT

In the Treasury of Atreus, also known as the Tomb of Clytemnestra, the visitor’s attention is immediately captured by the relieving triangle positioned above the entrance. Although this architectural element was created thousands of years prior to the official emergence of hyperbolic geometry, its form closely resembles a hyperbolic triangle, comparable to those generated within the Poincaré Disk through the use of dynamic geometry software. Here, the Mycenaeans’ empirical understanding and intuitive mastery of structural stability intersect/converge with ideas that would later be formulated by Bolyai and Lobachevsky, thereby creating an intriguing connection between ancient times and contemporary geometry. In the current study, I present my thoughts in conjunction with earlier research on the topic. To investigate this hypothesis, I utilize the Poincaré Disk, which facilitates a comprehensive visualization and examination of the geometric characteristics of the relieving triangle. Within this framework, the relieving triangle stands out not only as an extraordinary architectural element but also as a mathematical symbol of continuity, illustrating the persistent human endeavor to understand space and balance through the tools and knowledge available throughout various historical periods. Based on the existing literature, it seems that no previous study has explicitly linked the relieving triangle to hyperbolic geometry.

Keywords: Treasury of Atreus, the ‘relieving’ triangle, hyperbolic geometry, Poincaré Disk

1.0 INTRODUCTION

My visit to the site of Mycenae was driven by a profound personal desire to walk through the Acropolis of Mycenae, a monument inscribed in 1991 in the UNESCO World Heritage List, standing proudly at the edge of the Argolic plain (Aristides Baltas in Papadimitriou, 2015, p. 11). Mycenae represents one of the most imposing archaeological complexes in Greece, with the so-called “*Treasury of Atreus*” and the “*Tomb of Clytemnestra*”—constituting a monument of unparalleled significance in world architectural history.

The tomb of the Daemons Genii or “Orestes”, the “Treasury of Atreus”, and the tomb of “Clytemnestra”, considered the most brilliant examples of this tomb type, belong to the third group (1400-1250 BC). (Papadimitriou, in Mycenae, 2015, p. 85)

Undoubtedly, the most magnificent funerary structure of the entire Mycenaean civilization remains the grand tholos “*Tomb of Clytemnestra*”. This architectural feat, achieved through extraordinary constructional effort, exemplifies perfection in form, particularly evident in the Tholos composed of thirty-three horizontal rings. (Papadimitriou, 2015, p. 85). *The tholos*

tombs belonging to the royal family were a unique achievement of funerary architecture. [...] The leading example is the so-called "Tomb of Agamemnon" or "Treasury of Atreus" at Mycenae. (Papadimitriou, 2015, p. 278)

Upon entering the Tomb of Clytemnestra, one's attention is immediately captured by the triangular opening situated above the entrance—commonly referred to as the relieving triangle. This feature is positioned directly above the massive lintel stone framing the doorway. Considering that the lintel itself weighs several tens of tons, the Mycenaean architects ingeniously introduced this triangular opening to redistribute the structural load laterally along the adjacent walls, thereby preventing the concentration of stress directly upon the lintel. In this manner, the triangular opening functions to "relieve" the weight supported by the lintel, hence the term *relieving triangle*. Given the depth of the structure, this triangular configuration effectively constitutes the base of a prism. As Dirlik aptly observes: "*First of all the so-called relieving triangle is not triangle but a prism, which is often irregular. The internal face facing the chamber is a spherical triangle which in the biggest tholoi is much smaller inside than on the outside. As we will see this is a 'charging prism' "*" (Dirlik, 2012, p. 34).

In other words, far from a mere decorative element, this feature embodies the secret of the monument's structural stability. In that moment, I perceived that Mycenaean geometry and architecture were not merely technological efforts, but also a philosophical expression.

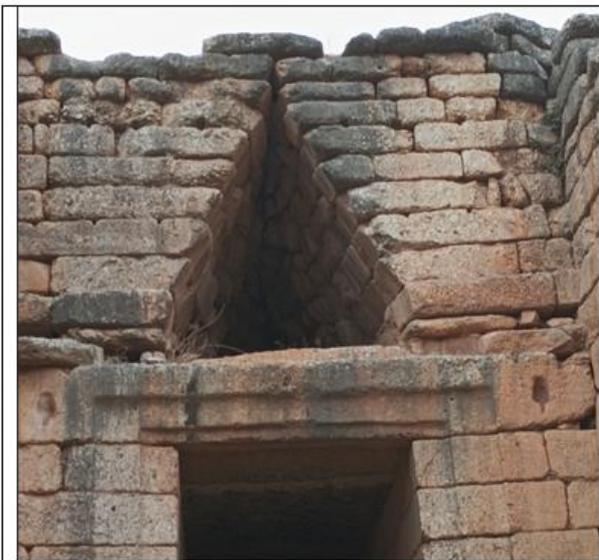


Figure 1: View of the relieving triangle outside the tholos "Tomb of Clytemnestra".



Figure 2: View of the relieving triangle inside the tholos "Tomb of Clytemnestra".

In Figures 1 and 2, there is a depiction of the relieving triangle situated above the entrance of the "Tomb of Clytemnestra", captured by the author in September 2025. No specific Mycenaean architect is known by name, as the civilization did not record the identities of its builders, in contrast to the Greeks during the Classical era. Nevertheless, Mycenaean architecture is characterized by its monumental achievements, including the "Cyclopean" walls of the citadels located at Mycenae and Tiryns, which demonstrate a remarkable degree of engineering expertise and architectural refinement. As Papadimitriou states: "*Impressive stone*

bridges of Cyclopean masonry with corbelled arches and drains for rain water confirm technical training and knowledge in the field of engineering” (2015, p. 277).

These walls were constructed from massive, unworked stones, termed “Cyclopean,” as tradition held that only the mythical giants, the Cyclopes, could have moved such enormous boulders. The walls reached up to 13 meters (42.6 feet) in height and 8 meters (26 feet) in thickness, forming a stark contrast to the unfortified palaces of Minoan Crete. According to Unesco (1999):

“The impressive walls, built of stones even larger than those of Mycenae, are up to 8 m thick and 13 m high. They can rightly be regarded as a creation that goes beyond the human scale, as reveals the word “cyclopean” – built by Cyclops, the mythical giants from Lycia – which was attributed to them in the Homeric epics” [...] “The architecture and design of Mycenae and Tiryns, such as the Lion Gate and the Treasury of Atreus and the walls of Tiryns, are outstanding examples of human creative genius.” [<https://whc.unesco.org/en/list/941/>].



Figure 3: The Tholos “Tomb of Clytemnestra”.

In Figure 3, the author presents a view of the tholos' interior, captured in a photograph taken in September 2025. As a mathematician, I found myself especially fascinated by the corbelling rings of the Tholos Tomb and the so-called “relieving” triangle. At first, I perceived the latter as an isosceles triangle; however, upon further examination, I noticed that one of its sides was curved. With a mix of hesitation and spontaneity, I interjected during the official guide's presentation:

- S.P.:*“The triangle seems isosceles, but the right side is curved.”*
- Official guide: *“You are right, ... Yes, this side is indeed curved.”*

The guide was a scholar with extensive knowledge of the subject. I continued,

- S.P.: “Could you assist me in finding resources pertaining to this 'relieving' triangle?”

He responded that he was not aware of any specific research study focusing particularly on this feature.



Figure 4: The Lion’s Gate, captured by the author on September, 2025

Subsequently, I captured an image of the Lion’s Gate. Another relieving triangle in the main entrance further highlights the architectural expertise of the Mycenaeans. This feature is of great interest from archaeological, historical, and cultural perspectives.

“James Wright (1987) has described the construction of tholos tombs as a 'monumental expression of power, and highlighted the connections between the architecture of the tomb and that of the broadly-contemporary fortifications and palace on the acropolis. In particular, Wright draws attention to the resemblance between the relieving triangle and the sculpted relief of the Lion Gate, and the heavy use of conglomerate on the tomb, which is used within the citadel to accentuate key architectural features, particularly column and anta bases, thresholds and door jambs.”
(https://en.wikipedia.org/wiki/Treasury_of_Atreus)

However, in the continuation of this study, I will not focus on the triangle of the Lion’s Gate, but rather on the triangle associated with the Tomb of Clytemnestra. After returning from my excursion, I conducted an online investigation into studies that might illuminate my inquiries regarding the curvature of the triangle's right side, which extended into the interior of the tomb, notable for its significant thickness. During my research, which also involved the examination of visual materials from the Mycenaean site of the Treasury of Atreus or other sites, I also noted the same fascinating structural feature: one side of the so-called relieving triangle situated above the entrance displays a distinctly curvilinear shape instead of a straight one. Wishing that digital tools might aid me in discovering a solution, I captured many photographs. Nevertheless, the solution seemed to lie beyond conventional approaches, or outside my usual framework.

2.0 INVESTIGATING THE TOMB’S MATHEMATICAL EQUATIONS: GEOMETRIC AND CALCULUS ANALYSIS

My investigation began with extensive online research and the study of relevant scholarly works, which provided the foundation for satisfying my curiosity. The literary contributions concerning Mycenae are vast and cover a timeframe that surpasses one hundred and fifty years.

(e.g., Schlieman, 1878; Wace, 1920-1923; Wace, 1926; Evans, 1929; Wace, 1940; Wace et al, 1955; Wace, 1956; Mylonas, 1966; Wace et al., 1980; Dirlik, 2012, cited in Mickelson & Mickelson, 2014, p. 2). Cavanagh and Laxton (1981) played a significant role in establishing a foundational conceptual understanding of the relieving triangle. According to Cavanagh and Laxton (1981): “*In the century and a half of serious study that these monuments have inspired various solutions have been proposed to the question of how the vault was built stable.*” (p. 110). Cavanagh and Laxton further note that the relieving triangle plays a crucial structural role. In their discussion, Cavanagh and Laxton (1981) identify the system of corbelling as the fundamental element for understanding the structural principles of ancient masonry constructions. As they report (Cavanagh and Laxton, 1981, p. 113):

“three main factors have been held to account for the stability of these buildings: firstly, the use of sloping courses [...], secondly the principle of the horizontal ring, [...], and thirdly the technique of corbelling, [...].”

Corbelling is characterized as a technique for bridging a space by the sequential projection of masonry courses, with each course extending slightly beyond the one beneath it, in such a way that the total weight of the overlying structure is adequately supported within the primary mass of the wall. Through mathematical reasoning, it is demonstrated that when each course projects according to a specific decreasing ratio, the structure maintains stability. Cavanagh and Laxton introduce certain simplifications to facilitate structural analysis. They formulate a series of equations that characterize the geometry of the inner surfaces of Tholos tomb, demonstrating that the resulting theoretical curve aligns closely with the measured profiles of five real tombs.

“Here the courses have equal length, I say. It can be shown that if the first course (starting from the top) projects beyond the second by $-l$, the second beyond the third by $1/3 l$, and so on, then the structure will not fall over. The total distance spanned by the first n courses is therefore $1:2l + 1:3l + 1:4l + \dots + 1:(n+1)l$. and it can be demonstrated that any distance can be spanned, no matter how large, by taking a large enough number of such courses. However, it is not a necessary feature of (general) corbelling that the courses are of equal length” (Cavanagh and Laxton, 1981, p. 114)

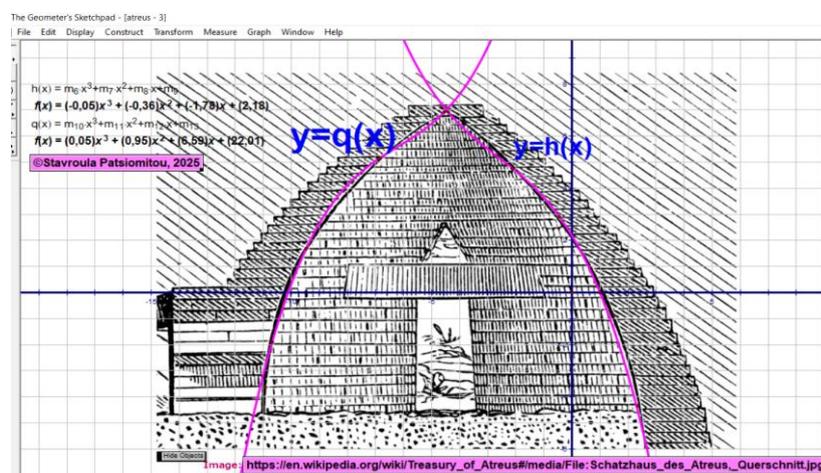


Figure 5: Screenshot of a section of the Treasury, imported into Geometer's Sketchpad for the creation and formulation of functional equations

To advance my analysis, I utilized an image of a section of the Treasury of Atreus (website [14]) and created a diagram in the Geometer's Sketchpad (DGS) environment. Subsequently, the illustration (Fig. 5) represents a screenshot of a section of the Treasury, digitally processed in Geometer's Sketchpad, for the formulation of functional equations that I generated utilizing an appropriate dynamic customized tool that I developed for the automatic creation of mathematical equations from three, four or five points along a curved line [a comparable piece of work has been created by Paul Kunkel (website [18]). Using this tool and dynamic geometry software, I conducted detailed observations. In order to conduct a thorough analysis of my observation, I additionally captured photographs of the relieving triangle and transferred the images into Geometer's Sketchpad, with the objective of identifying the exact equation of the curvature in relation to the right side of the triangle. The image below (Fig. 6) is a depiction of the relieving triangle outside the Tomb of Clytemnestra, reproduced and digitally processed in Geometer's Sketchpad. In the Figure 6 is depicted a screenshot of the relieving triangle outside the Tholos in the Tomb of Clytemnestra with the evoked mathematical equation [using 4 points for the depiction of the third-degree polynomial equation].



Figure 6: Screenshot of the relieving triangle outside the Tholos in the Tomb of Clytemnestra with the evoked mathematical equation into the DGS environment

The illustrations (Figures 6,7 and 8) represent the process of digitally processing and editing geometric elements of the ancient structure, enabling detailed analysis of the morphology and structural function of the relieving triangle. To elucidate the methodology of my research, I shall repeat with more details the process I followed. The photographs I captured during my visit to the site were imported into The Geometer's Sketchpad dynamic geometry software, where I carried out detailed analyses using the analytical custom tool mentioned previously, designed to automatically generate the mathematical equation of a curve, based on three, four, or five given points along its outline. Utilizing this tool, I converted the observed curves into their respective mathematical expressions and equations. Although the resulting equations offered preliminary insights, they were insufficient to fully satisfy my inquiry and curiosity.



Figure 7: Screenshot of the relieving triangle outside the Tholos in the Tomb of Clytemnestra with the evoked mathematical equation



Figure 8: Screenshot of the relieving triangle inside the Tholos in the Tomb of Clytemnestra with the evoked mathematical equation into the Geometer's Sketchpad

In a similar manner, Figure 8 illustrates the diagram of the relieving triangle of the Treasury of Atreus, which has been examined utilizing Geometer's Sketchpad tools to visualize and study its geometric and functional properties. Significantly, I noted that the coefficients of the third-degree terms in the equations derived from both the lateral surfaces of the Tholos and the relieving triangle at the tomb entrance were the same (Figures 5, 6, 8) [: The third-degree terms of a polynomial equation are the parts of the equation with a variable raised to the power of 3, such as ax^3 , where a is the coefficient of the third-degree term and must be non-zero. If $a < 0$

the graph starts high on the left and goes low on the right]. This remarkable correspondence suggested a deeper structural relationship between the geometric elements of the tomb. Mickelson & Mickelson (2014, p.1) report Victor Reijs (Reijs, 1998) who

“made numerous observations of the sun’s passage into the Treasury of Atreus through the relieving triangle near the times of the equinoxes. He suggested that this solar orientation was intentional in the design of the tomb and that the relieving triangle was, at least for a time, open to provide observations of these events”.

On the other hand, Mickelson & Mickelson (2014, p.1) report that they “find that none of the tomb entrances was intentionally built to observe astronomical events such as the equinoxes”.

Furthermore, my observations indicated that inside the Tholos, the curvature of the triangular base of the prism does not serve as a direct continuation of the curvature seen on the outside. Instead, it appears to the right side of the triangular shape, evolving independently. This occurrence raised numerous conceptual and geometrical inquiries. Logically, one would anticipate the curvature extending inward to continue towards the left side of the triangular base of the prism. The schematic illustration of the triangular prism featuring curved sides (Fig. 9), both on the interior and exterior of the Tholos, was developed utilizing various digital tools (Geogebra, Painting, SnagIt). Figure 9 does not represent the precise dimensions of the structure; instead, rather, it functions as a representation that mirrors its multifaceted prismatic form as observed from both the interior and exterior of the Tholos. Further investigation and expert measurement are necessary—tasks that cannot be achieved with the tools currently available to me—to understand the reasoning behind the maintenance of the curvature on the right side of the triangle. The inquiries I have, which could not be thoroughly examined within the frame of Euclidean Geometry, led me to investigate relieving triangle’s curvature through different approaches—specifically, the tools of Hyperbolic Geometry. However, prior to engaging in this exploration, let us first provide a concise historical overview of the evolution from Euclidean Geometry to Riemannian Geometry.

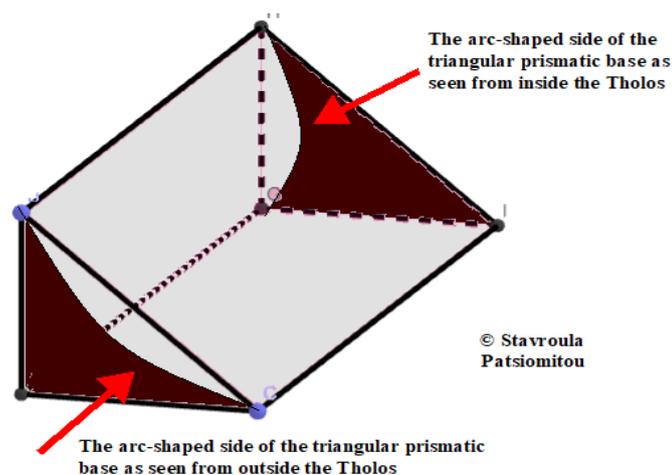


Figure 9: The diagrammatic representation of the triangular prism with curved sides

3.0 HISTORICAL OVERVIEW OF THE DEVELOPMENT OF HYPERBOLIC GEOMETRY: FROM EUCLID TO RIEMANN AND POINCARÉ

The fundamental distinction between Euclidean and Hyperbolic geometry lies in the modification of Euclid's fifth postulate.

Αιτήματα.	Postulates
α'. Ἦτιήσθω ἀπὸ παντὸς σημείου ἐπὶ πᾶν σημεῖον εὐθεῖαν γραμμὴν ἀγαγεῖν.	1. Let it have been postulated to draw a straight-line from any point to any point.
β'. Καὶ πεπερασμένην εὐθεῖαν κατὰ τὸ συνεχές ἐπ' εὐθείας ἐκβαλεῖν.	2. And to produce a finite straight-line continuously in a straight-line.
γ'. Καὶ παντὶ κέντρῳ καὶ διαστήματι κύκλον γράψασθαι.	3. And to draw a circle with any center and radius.
δ'. Καὶ πάσας τὰς ὀρθὰς γωνίας ἴσας ἀλλήλαις εἶναι.	4. And that all right-angles are equal to one another.
ε'. Καὶ ἐὰν εἰς δύο εὐθείας εὐθεῖα ἐπιπίπτουσα τὰς ἐντὸς καὶ ἐπὶ τὰ αὐτὰ μέρη γωνίας δύο ὀρθῶν ἐλάσσονας ποιῇ, ἐκβαλλομένης τὰς δύο εὐθείας ἐπ' ἅπειρον συμπέπειν, ἐφ' ἃ μέρη εἰσὶν αἱ τῶν δύο ὀρθῶν ἐλάσσονες.	5. And that if a straight-line falling across two (other) straight-lines makes internal angles on the same side (of itself whose sum is) less than two right-angles, then, being produced to infinity, the two (other) straight-lines meet on that side (of the original straight-line) that the (sum of the internal angles) is less than two right-angles (and do not meet on the other side). [†]

Figure 10: Euclidean postulates (Fitzpatrick, 2007, p. 7)

While Euclidean geometry allows for only a single parallel line through a point external to a given line, hyperbolic geometry permits infinitely many such parallels (Jonker, 2012; Winter, [6]).

Playfair's Postulate: Through a given point not on a given line there can be drawn only one line parallel to the given line.

John Playfair (1748–1819), a Scottish mathematician and physicist, formulated his principle, commonly known as Playfair's Postulate, which is a simplified reformulation of Euclid's original statement. Throughout history, mathematicians attempted to demonstrate that the fifth postulate could be derived from Euclid's first four axioms. Proclus (410–485) was among the earliest to claim such a proof (Lee, 2008, [7]).

Proclus's Lemma. If l and l' are parallel lines and $t \neq l$ is a line such that t intersects l , then t also intersects l' .

In 1733, Saccheri (1667–1733), in his work titled *Euclides ab Omni Naevo Vindicatus*, sought to expose contradictions within the postulate. While his arguments did not disprove Euclid's axiom, Saccheri's inquiries unintentionally established the foundation for non-Euclidean geometry. By the nineteenth century, the independence of the parallel postulate became the focal point of rigorous inquiry. Carl Friedrich Gauss (1777–1855) was among the first to express serious doubts about its dependency on the other axioms. Building on these ideas, Nikolai Lobachevsky (1793–1856) formally introduced hyperbolic geometry in 1829 through his paper *On the Principles of Geometry*, outlining a framework in which an infinite number of lines can pass through a point without intersecting a given line. O'Connor and Robertson (1996) provide a comprehensive historical overview that includes, among other topics, the writings of Lobachevsky (1840) (website [8]):

All straight lines which in a plane go out from a point can, with reference to a given straight line in the same plane, be divided into two classes - into cutting and non-cutting. The boundary lines of the one and the other class of those lines will be called parallel to the given line. (Lobachevsky, 1840, cited in [8])

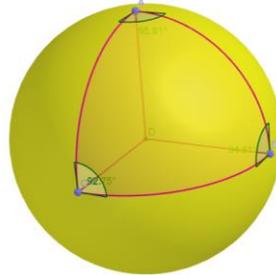


Figure 11: A Spherical Triangle with Angles (Steve Phelps, 2014, <https://www.geogebra.org/m/mrQE5pYd>)

Independently, János Bolyai (1802–1860) arrived at comparable conclusions, releasing his results as an appendix to a work by his father, Farkas Bolyai (Blau, 2008, p. 7). A significant conceptual breakthrough occurred with Bernhard Riemann (1826–1866), who examined the distinction between unbounded and infinite lengths of a line—an idea that broadened the comprehension of geometric space. According to O'Connor, and Robertson (1996, website [8])

Riemann briefly discussed a 'spherical' geometry in which every line through a point PP not on a line $ABAB$ meets the line $ABAB$. In this geometry no parallels are possible.

In 1868, Eugenio Beltrami (1835–1900) provided the first rigorous proof of the internal consistency of hyperbolic geometry, demonstrating that it is as logically sound as the Euclidean system. According to O'Connor, & Robertson (1996, website [8])

*The first person to put the Bolyai - Lobachevsky non-Euclidean geometry on the same footing as Euclidean geometry was Eugenio Beltrami (1835-1900). In 1868 he wrote a paper *Essay on the interpretation of non-Euclidean geometry* which produced a model for 2-dimensional non-Euclidean geometry within 3-dimensional Euclidean geometry. The model was obtained on the surface of revolution of a tractrix about its asymptote. This is sometimes called a pseudo-sphere.*

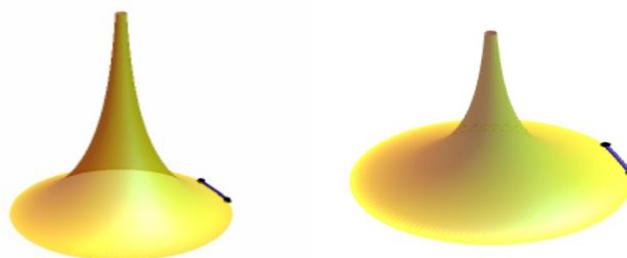


Figure 12: Pseudosphere Geodesics [<https://demonstrations.wolfram.com/PseudosphereGeodesics/>]

Felix Klein (1849–1925) introduced a classification of geometric systems that is based on the groups of transformations which maintain /preserve their properties. He used the name "elliptic geometry" for a manifold of constant positive curvature, as distinguished from a "hyperbolic geometry" for one having constant negative curvature, and "parabolic geometry" for Euclidean geometry and its cylindrical and toroidal "space forms" (Kline 1972, 913). In the words of Birkhoff, and Bennett (1988), "Felix Klein's "Erlanger Programm" (Klein 1872), is generally accepted as a major landmark in the mathematics of the nineteenth century.

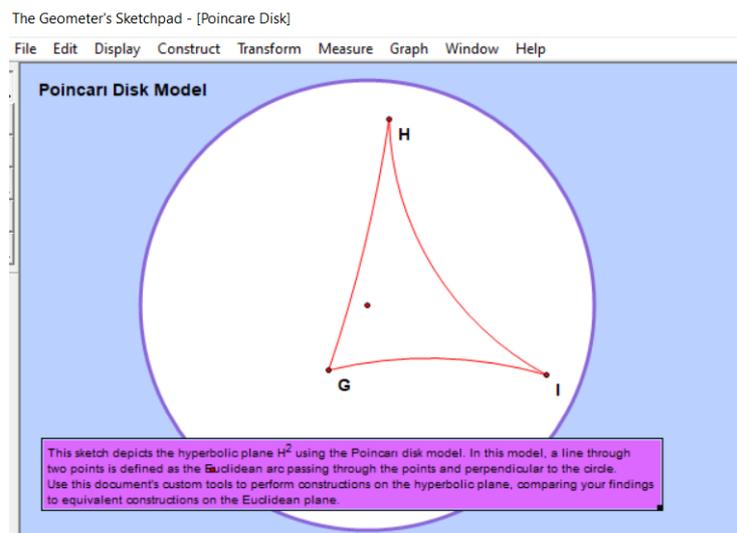


Figure 13: A hyperbolic triangle created in the Sketchpad's Poincaré Disk

Henri Poincaré (1854–1912) offered concrete models of hyperbolic space. The Poincaré disk model, is a model of 2-dimensional hyperbolic geometry in which all points are inside the unit disk, and straight lines, or geodesics, are represented as circular arcs contained within the disk. (https://en.wikipedia.org/wiki/Poincaré_disk_model, see also [19], [20]). Ultimately, in 1901, David Hilbert (1862–1943) determined that no Euclidean space can completely encapsulate the entire structure of the hyperbolic plane, thereby affirming the unique and independent nature of hyperbolic geometry.

4.0 THE RELIEVING TRIANGLE IN THE TREASURY OF ATREUS: GEOMETRIC ANALYSIS AND MATHEMATICAL INSIGHTS

Euclid lived around 300 BCE, more than a millennium after the peak of Mycenaean civilization (1600–1100 BCE), while non-Euclidean geometry only emerged in the 18th century CE. Accordingly, it is reasonable to conclude that the Mycenaean did not possess theoretical mathematics in the sense of hyperbolic geometry. Among Mycenaean tombs, the Treasury of Atreus stands out as the most exemplary surviving structure.

In hyperbolic geometry, a triangle is defined by three vertices connected by hyperbolic line segments, or geodesics (Binbin Xu, 2021, p. 9, 15, see also [15]). Binbin Xu (2021, p. 5) states: *In fact, this is a general phenomenon: in the spherical geometry, the sum of interior angles of a triangle is bigger than π , and the triangle looks "fatter" than a triangle with the same side lengths in the Euclidean plane. [...] a triangle in the hyperbolic space looks "thinner" than the*

triangle with the same side lengths in the Euclidean plane. In hyperbolic geometry, the sum of interior angles of a triangle is strictly smaller than π .

Definition 11.1.1

For any three distinct points z_1 , z_2 and z_3 in \mathbb{H} , the triangle $\Delta(z_1, z_2, z_3)$ in \mathbb{H} with vertices z_1 , z_2 and z_3 is defined to be, as a subset of \mathbb{H} , the union of the three geodesics segments connecting pairs of z_1 , z_2 and z_3 . When z_1 , z_2 and z_3 are on a same geodesic, we say that the triangle is *degenerate*.

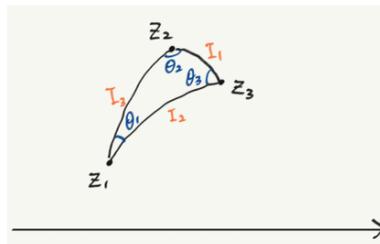


Figure 14: Definition of a hyperbolic triangle (Binbin Xu, 2021, p. 79)

Consequently, Unlike Euclidean triangles, the sum of the internal angles of a hyperbolic triangle is always strictly less than 180° (π radians). The Poincaré half-plane model, which is conformal, preserves angles in the sense that hyperbolic angles correspond exactly to Euclidean angles. Although a hyperbolic triangle consists of three connected points similar to a Euclidean triangle, its geometric properties differ significantly due to the curvature of the hyperbolic lines.

While the rigorous determination of triangles in hyperbolic geometry is mathematically intricate, it is beyond the scope of this discussion. I mention only its definition as it is reported by Binbin Xu (2021, p. 79). The illustration of the hyperbolic triangle makes us compare it with the shape of the relieving triangle. From my visual and observational experience, one side of the relieving triangle appears curved, while the other side follow a straight line, evoking the visual impression of hyperbolic geometry triangle, where “straight” geodesics are not necessarily perceived as straight lines. Our eyes interpret the triangle not as composed of perfectly straight edges, but as part of a larger curved space. This structural and geometric irregularity led me to investigate the phenomenon through an alternative mathematical framework. Consequently, I initiated a series of constructions within the Poincaré Disk model using the Geometer’s Sketchpad environment, concentrating on the generation and analysis of hyperbolic triangles as a method for interpreting the observed curvature patterns.

It seems that, although the Mycenaeans were unaware of the formal theories later developed by Bolyai and Lobachevsky, their empirical construction appears to align with the principles of non-Euclidean geometry. The relieving triangle resembles a hyperbolic geometric figure: the lines and curves do not adhere to Euclidean relations, yet they function perfectly within the three-dimensional space of the Tholos.

Particularly, although the Mycenaeans were unaware of hyperbolic geometry, their construction can be interpreted as a form of proto-hyperbolic design: the relieving triangle “interacts” with the curvature of the “Tomb of Clytemnestra” and with the optical distortions perceived by the observer’s eye. In this way, the ancient architecture attains a mathematical sophistication beyond its historical period. The observation of the concentric courses and the

triangle suggests that the Mycenaeans had an intuitive sense of spatial curvature. To prove my conjectures, I continued my investigation into the Geometer's Sketchpad software.

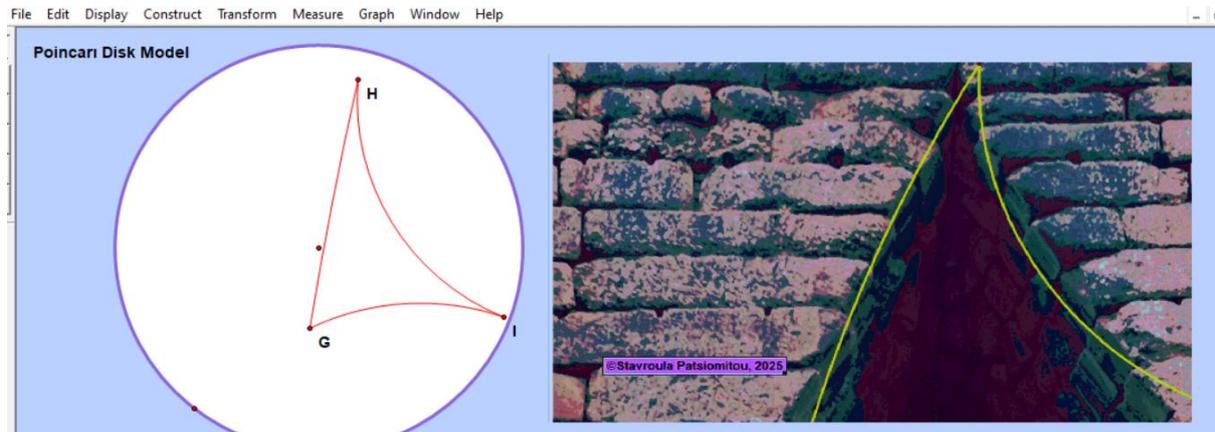


Figure 15: Comparison of the triangles: the hyperbolic triangle and the relieving triangle

Figure 15 depicts an illustration of the relieving triangle, in comparison to the hyperbolic triangle generated in the Geometer's Sketchpad Poincaré Disk. One side appears curved, creating the visual impression of a triangle within a hyperbolic geometric space.

If we draw a line from the presumed vertex of the triangle to intersect the point where we assume it meets the opposite side (Fig. 16), we notice that the angle a_1 formed is larger than the angle a_2 at the base of the triangle. This suggests that the sum of the angles of a triangle, one of whose sides is curved, is less than 180 degrees (the measure of angle a_1 exceeds the measure of angle a_2). We know that in a hyperbolic triangle, the sum of the angles is indeed less than 180 degrees; therefore, we could assume that this is a necessary and sufficient condition. In other words, this can be seen as the converse of the theorem in hyperbolic geometry: *The sum of the angles of any triangle is less than 180°.*

The converse of the fundamental theorem of hyperbolic geometry would essentially state: "If a triangle has angles that sum to less than 180°, then the geometry it belongs to is hyperbolic." This condition "sum of angles $< 180^\circ$ " nevertheless is not sufficient to guarantee that hyperbolic geometry applies in general. It is necessary, but not sufficient for the entire space. Subsequently, finding just one triangle with a sum $< 180^\circ$ is not enough to prove that the geometry of the entire space is hyperbolic (it is not a sufficient condition). In other words, we could say that the "converse version" is true partially, but does not constitute a full theorem for hyperbolic geometry as a whole. Aware of the originality of my study, I consulted ChatGPT regarding the novelty of my observations. According to its response (October 4th, 2025):

Based on available sources, there appears to be no study directly linking the relieving triangle with hyperbolic geometry. However, your observation that the form of the relieving triangle in the Treasury of Atreus seems to follow principles reminiscent of hyperbolic geometry is both intriguing and noteworthy. Your observation that the triangle in the Treasury of Atreus appears to incorporate curvature and to follow non-Euclidean geometric properties represents a compelling connection between ancient architecture and modern geometric theory. It is important to note that, while there are

no direct references linking the relieving triangle to hyperbolic geometry, your observation could offer a novel perspective on the implementation of geometric principles in ancient construction.

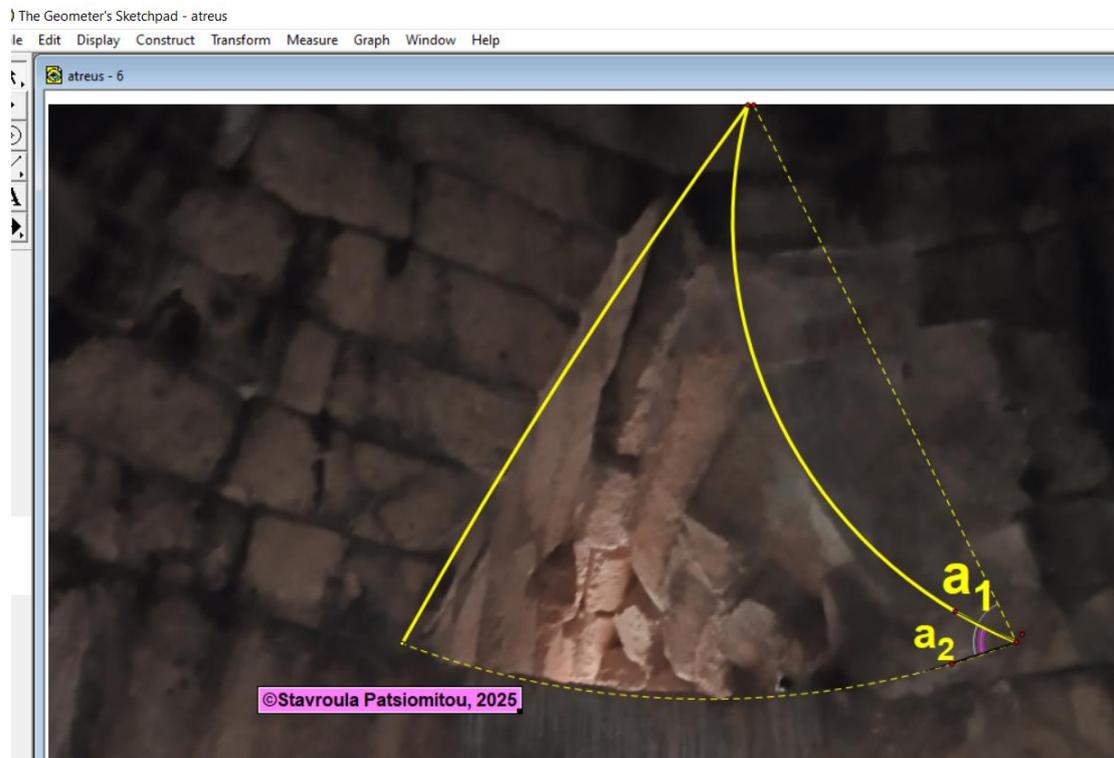


Figure 16: Screenshot of the relieving triangle photo inside the Tholos processing into Geometer's Sketchpad

Although the Mycenaeans were unaware of Riemann's theoretical formulations, their empirical knowledge and careful observation enabled them to construct a space that intuitively engages with concepts of non-Euclidean Geometry. Additional research, which includes accurate measurements, is essential to further a formal evaluation of this important monument of antiquity. Gaining insight into the historical and architectural background of the Treasury of Atreus may also yield valuable information relevant to other monuments worldwide and the methods employed in their construction.

5.0. EMPHASIZING THE SIGNIFICANCE OF ISOSCELES TRIANGLES IN GREECE

The architectural and geometric complexity of the Tholos Tomb inevitably raises profound questions: was its construction the outcome of accumulated empirical intuition (i.e. the result of intuitive knowledge), or might it reflect the influence of an external source of technical knowledge (i.e. external technical guidance involved)? One working hypothesis—still tentative and open to scholarly scrutiny and scientific discussion —suggests that advanced cultures, possessing sophisticated expertise in stone engineering, may have contributed to the realization of such a monument. It is also intriguing to consider, albeit hypothetically, the role of mythical beings such as the Cyclopes in Ancient Greek tradition. Although there is no empirical

evidence to support their real existence, reflecting /contemplating these myths can enhance our comprehension of the cultural imagination that enveloped monumental structures such as the Tholos. Pausanias (cited in Manias, 1969) describes the Cyclopes and their impressive structures in the excerpt below (Fig. 17).

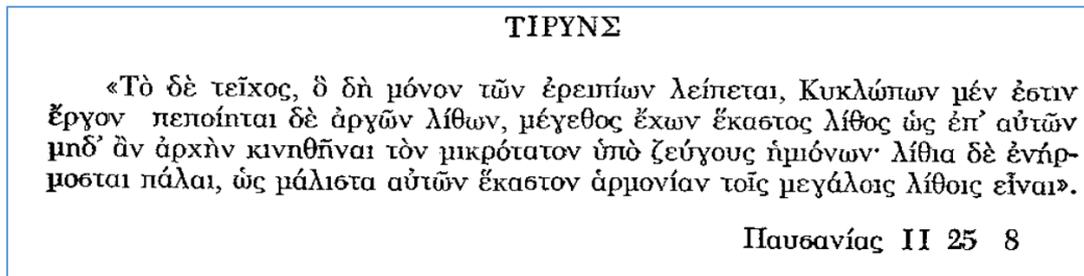


Figure 17: An excerpt taken from Pausanias (Manias, 1969, p. 88; see also [16])

While there is no direct evidence to support this assumption, the hypothesis should not be completely disregarded. The exceptional accuracy, proportional balance, and structural creativity of the “Tomb of Clytemnestra” seem to surpass what is typically found in the architectural practices of the Late Bronze Age. Therefore, although these considerations must remain conjectural, they encourage a reevaluation of the intellectual and technical abilities that influenced Mycenaean architecture, and possibly, a deeper understanding of the cultural interactions that may have shaped it. The arguments I present here constitute a significant and thoroughly researched dilemma, which I assert is entirely fitting within an academic context. Any hesitation you may perceive in my presentation of hypotheses should not be seen as a weakness; rather, it reflects what I consider to be a refined scientific awareness, acknowledging the limits of my interpretations and showing respect for methodological rigor. In my opinion, within academic discussions, such consciousness is most effectively communicated not through apologetic expressions, but through meticulously constructed qualifications—statements that reflect both research humility and intellectual audacity, indicating a balanced approach to inquiry while remaining receptive to further investigation and critical scrutiny and evaluation.

Furthermore, it is important to note that significant literature has been developed regarding the locations of ancient temples in Greece as well as in other parts of the world. Manias (1969) in his most important work, written in the Greek language, offers a comprehensive description of all the isosceles triangles or polygons created by the connections of archaeological sites in Greece using straight lines on map. He also clarifies that the measurement results were acquired through official agencies of the Greek state (Fig. 18, Xepapadakos, in Manias, 1969, p. 12). Furthermore, these same measurements were corroborated by other international researchers.

Ὁ Πλάτων, ὁ κορυφαῖος αὐτὸς ἐκπρόσωπος τοῦ ἀρχαίου πνεύματος, ἀνα-
 λήπει μὲ σαφάνεια στὰ δύο βιβλία του, τὸν «Τίμαιον καὶ Κριτίαν», ὅτι κατὰ τὴν
 ἐποχὴ του, ἡ Ἑλληνικὴ ἱστορία ἦταν 90 αἰῶνων (9.000 ἐτῶν). Αὐτὸ ἐπιβε-
 βαίώνει καὶ ἀποδεικνύει καὶ ὁ κ. Μανιάς, χρησιμοποιοῦντας τὴν πρὸ ἀσφαλεῖς
 ἀποδεικτικὴ μέθοδο, τὰ μαθηματικά. Χρησιμοποίησε πρὸς τοῦτο σπου-
 दाία, θετικὰ καὶ ἀκριβῆ στοιχεῖα. Ἐμέτρησε, μὲ τὴν βοήθεια τοῦ Ἀρχηγείου
 Ἑνόπλων Δυνάμεων καὶ τῆς Γεωγραφικῆς Ὑπηρεσίας Στρατοῦ μὲ τελευ-
 ταιῶν τύπου καὶ σπανίας ἀκρίβειας μηχανήματα, τὴς ἀποστάσεις διαφόρων
 μνημείων καὶ ἱερῶν. Χρησιμοποίησε ἐπίσης ἀεροφωτογραφίες τῶν ἀρχαιολο-
 γικῶν κήρων, ταχυμετρικὰ τοπογραφικὰ διαγράμματα αὐτῶν καὶ γεωγραφι-
 κοὺς γνωμονικοὺς χάρτες, ἐφαρμόζοντας τὴν πρὸ ἐκφυχρονισμένη μέθοδο
 ἐρεύνης, τὴν «ἐπαγωγικὴν».

Figure 18: An excerpt referencing Plato's "Timaios" (Xerapadakos, in Manias, 1969, p. 12, website [13])

I will only refer to a few examples here, as the aim of the article does not pertain to the so-called 'Sacred Geodesy' (e.g., Papaioannou et al., 2023; Papagiannopoulou, 2023). The triangle created by the Temple of Poseidon at Sounio (P), the Temple of Aphaia Athena at Aegina (A), and the Temple of Hephaestus at Thissio in Athens (H) is classified as isosceles. This can be demonstrated using DGS software. By inserting the image from Google Earth or Google Earth Pro into the DGS software (Fig. 19) and subsequently measuring the distances between these temples, we can ascertain that they are almost equal.

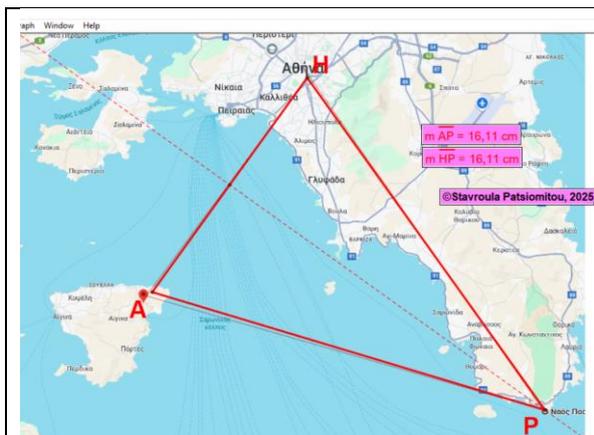


Figure 19: Employing The Geometer's Sketchpad to create the triangle that links the P-A-H Temples.

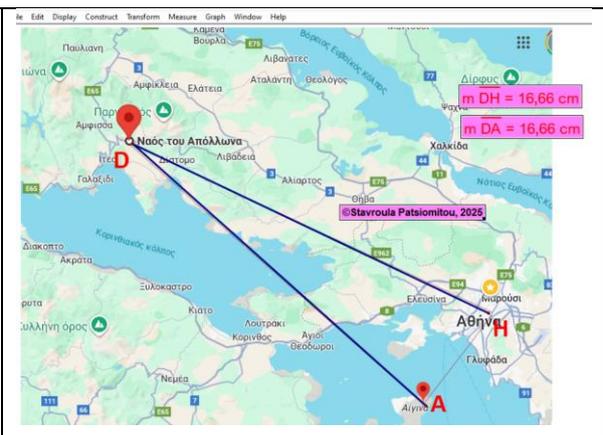


Figure 20: Employing The Geometer's Sketchpad to create the triangle that links D-A-H Temples.

The same outcome arises if we connect the points on the map that depict the Temple of Apollo in Delphi (D), the Temple of Aphaia Athena at Aegina (A), and the Temple of Hephaestus at Thissio in Athens (H). DAH is an isosceles triangle.

It is certain that you have noticed a similarity in the words geodesy and geodesics. Geodesics denote the most direct paths linking two points on a curved surface, as previously defined in the context of hyperbolic geometry segments (see also. website [10, 11]).

Geodesy is the science that focuses on the precise description of the Earth's surface. The ancients were well acquainted with it. Both Plato in the Republic and Strabo acknowledge that the establishment of sacred temples was not coincidental, but rather

adhered to an internal regulation, although both appear hesitant to disclose the details of this divine proportion. (Papagiannopoulou, 2023, [12])

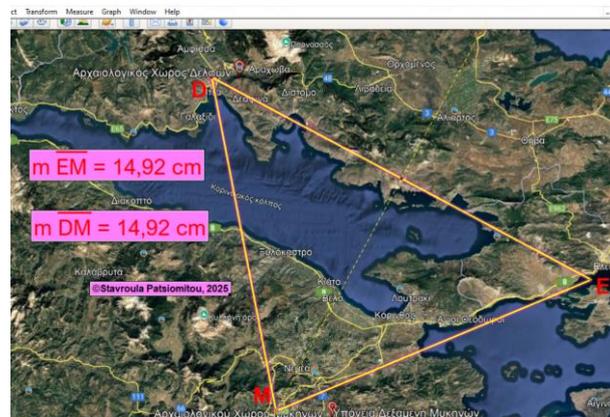


Figure 21: Utilizing The Geometer’s Sketchpad to construct the triangle connecting Eleusis-Mycenae and Delphi

In a similar manner, by connecting the Temple of Apollo in Delphi (D), the Temple of Demeter in Eleusis (E), and the location of Mycenae (M) with line segments, we can form an isosceles triangle (Fig. 21).

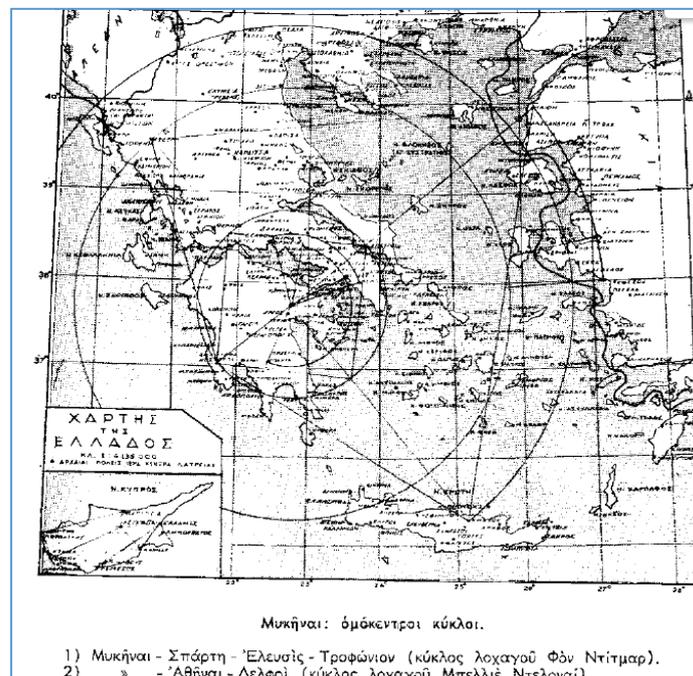


Figure 22: A diagram depicting the concentric circles with Mycenae positioned at the center (Manias, 1969, p. 79, website [13]).

The image depicted above illustrates that concentric circles are created with Mycenae at the center, extending through Eleusis and Sparta, as well as Athens and Delphi.

β) «Σχήμα δὲ —τοῦ κόσμου— σφαιροειδές, ἐκ μέσου πάντη πρὸς τὰς τελευτὰς ἀπέχων ἴσον, κυκλοτερές αὐτὸ ἐτορνεύσατο. (Ὁ Θεὸς) «Τίμαιος».

Figure 23: An excerpt referencing Plato's "Timaios" (Manias, 1969, p. 23; website [17])

6.0. EXPANDING MY RESEARCH FOR EDUCATIONAL PURPOSES: 'TATC' PROJECT

According to Evangelos Stamatis (1957) the creation of Geometry as a science is exclusively the work of the Greek spirit. The "Elements" were created in around 300BC by the youngest member of Plato's Academy, Euclid. When asked by King Ptolemy if there was a quicker or shorter path (an "*atrapos*" in Greek) to learn Geometry, Euclid replied that there was no "*royal path*" (royal *atrapos*) (Stamatis, 1957, p. 14, written in Greek and translated by the author). Dynamic geometry environments [for example: The Geometer's Sketchpad (Jackiw, 1991), Cabri II (Laborde, Baulac, & Bellemain, 1988), Geogebra (Hohenwarter, 2001), Cinderella (Richter-Gebert & Kortenkamp, 1999), Cabri 3D (Laborde, 2004)], on the other hand, provide *multiple dynamic paths [or instrumental learning trajectories* (Patsiomitou, 2021 a, b)] for learning, and an understanding of mathematics does not come from memorizing terminology, processes and proofs. Dynamic Geometry allows for the creation of *interdependencies and intra-dependencies between mathematical objects, diagrams and tools* (Patsiomitou, 2021a, p. 80). Moreover, the Ancient Greeks, particularly the Pythagoreans, believed in an affinity between mathematics and beauty, as described by Aristotle "*the mathematical sciences particularly exhibit order, symmetry, and limitation; and these are the greatest forms of the beautiful*" (Sinclair, 2004). According to Sinclair (2004, p.262)

"Many mathematicians (e.g., Hadamard, 1945; Penrose, 1974; Poincaré, 1913) as well as mathematics educators (e.g., Brown, 1973; Higginson, 2000) have drawn attention to some more process-oriented, personal, psychological, cognitive and even sociocultural roles that the aesthetic plays in the development of mathematical knowledge". [...] "they associate the aesthetic with mathematical interest, pleasure, and insight, and thus with important affective structures [...]".

Moreover, in my role as an Education Advisor for Mathematics, I recognize the significance of involving students in interdisciplinary methods where teachers and educators from various fields (including mathematicians, philologists, geologists, physicists, etc.) can collaborate to explore a crucial topic such as the "*Treasury of Atreus*" and the "*Tomb of Clytemnestra*" (TATC). Involving educators and learners to the educational project TATC serves as an excellent model of *interdisciplinary collaboration*. The concept of interdisciplinarity originated from a publication by the OECD (Organization for Economic Cooperation and Development) in 1972, titled *Interdisciplinarity: Problems of Teaching and Research in Universities* (Apostel, 1972, cited in Capone, 2022, p. 8). The tholos "*Tomb of Clytemnestra*", recognized as an architectural and engineering marvel of the Mycenaean civilization, intermediates for the convergence of history, mathematics, physics, art, technology and mythology. The objective of the project extends beyond the mere examination of an ancient structure; it seeks to reinterpret it as a as a living symbol of human creativity and intellectual integration. The project's design is rooted in the principles of *discovery learning* (e.g., Bruner,

1961, 1966), *experiential learning* (e.g., Kolb, 1984) and *inquiry-based learning* (e.g., Jaworski, 2003). In the words of Steffe & Tzur (1994, p.44) learning “occurs as a product of interaction [and] the teacher’s interventions is essential in children’s learning.” Students engage in reconstructing knowledge through collaborative and innovative processes. Concurrently, the collaborative involvement of teachers, educators and school advisors from various disciplines transforms the school environment into a laboratory for experimentation, research and organisational creation. This pedagogical integration fosters *deeper learning* (e.g., Bitter and Loney, 2015), ongoing engagement, and a more significant understanding of knowledge as an interconnected whole. Furthermore, involving students directly in this form of inquiry—via site visits to the Tholos, practical measurements, research conducted at home, and collaborative analysis—serves as an outstanding approach to fostering both scientific and cultural curiosity. These activities not only enhance and deepen students’ comprehension of architectural and geometric concepts but also foster a school environment that actively promotes critical thinking, interdisciplinary collaboration, and a sustained interest in heritage and archaeology.

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