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FROM THE CONCEPT OF SCHEMA TO THE IDEA OF "INSTRUMENTAL" SOCIAL SCHEMA

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ABSTRACT

This paper aims to investigate and discuss the concept of schema within various theoretical frameworks, highlighting its essential importance for comprehending mathematical contexts. The article discusses Bartlett, recognized as the pioneer in introducing the concepts of "schema" and "schemata" in his research on memory in the field of cognitive psychology. Additionally, it emphasizes Skemp's important contributions to the comprehension of schema. Furthermore, the psychological dimension of schema is clarified by Piaget's theory and plays a crucial role in Vergnaud's theory of conceptual fields. During the process of instrumental genesis, individuals, such as students in a classroom setting, actively develop what Rabardel describes as "utilization scheme" or "usage scheme" related to a tool. Rabardel posits that the application of tools encompasses both personal and social aspects. Consequently, social schemes emerge through interpersonal interactions and are shaped by the utilization of tools and artifacts within a given context. From this perspective, schemas contain social components. The incorporation of blended teaching methods, along with digital technologies and artificial intelligence, has significantly transformed the practices utilized in academic and educational organizations and institutions. Consequently, learners and teachers develop "instrumental" schemata, which suggests that these schemata are not static but rather dynamic, evolving continuously within a digital/instrumental/AI context. This transformation has also influenced the structure of various activities included in student textbooks.

Keywords: schema, utilization schemes, "dynamic" schema, instrumental social schema

1.0 INTRODUCTION: What is a "schema"?

The study of learning processes is fundamentally connected to the evolution of cognitive psychology, which has primarily concentrated on child development and, more generally, the growth of living beings. The concept of schema (with the plural forms being schemata or schemas) holds significant importance in the field of cognitive psychology. Its origins can be traced back to ancient Greece, where it was considered in a more general context rather than the specific framework utilized in cognitive psychology. According to Corcoran & Hamid (2016):

> A *schema* (plural: *schemata*, or *schemas*), also known as a *scheme* (plural: *schemes*), is a linguistic "template", "frame", or "pattern" together with a rule for using it to specify a potentially infinite multitude of phrases, sentences, or arguments, which are called *instances* of the schema. […]. The Greek word 'schema'; was used in Plato's Academy for "[geometric] figure" and in Aristotle's Lyceum for "[syllogistic] figure". Although

> Aristotle's syllogistic figures or "schemata" were not schemas in the modern sense, Aristotle's moods were.

Bartlett (1932) introduced the term "schema and schemata" when he was investigating memory in the field of cognitive psychology. According to Bartlett (1932) schema is "*an active organization of past reactions or experiences*" (p. 201). He used this term to interpret the structural organization of knowledge, within which every new piece of knowledge and experience must be related to something we already know: to understand something, it is necessary to assimilate it into an appropriate schema that pre-exists in our memory. Bartlett's observation of how a student recalls information led him to conclude that this depends on the student's cognitive structures and the way they have encoded the information in relation to their prior knowledge. Schemas represent cognitive frameworks that structure knowledge and influence behaviour (Piaget, 1936/1952). They serve as fundamental components of cognitive growth. Through their engagement with the environment, children create and adjust their schemas to enhance their comprehension of the surrounding world. Schemas (or schemata) serve as units of comprehension that can be organized hierarchically and interconnected in intricate relationships with one another. Cognitive psychology has considered the human mind as an information-processing machine. Rumelhart (1980) defined schema as "*a data structure for representing the genetic concepts stored in memory*"(p.34). Consequently, schemas or schemata encapsulate both declarative knowledge, which pertains to what an individual knows and the factors influencing their learning process, and procedural knowledge, which relates to the application of specific cognitive skills. Skemp (1971, as referenced in Hitt, 2002, p. 243) asserts in his work "The Psychology of Learning Mathematics" that comprehension is closely linked to the alignment of new information with the pre-existing schemas held by the individual. He states:

> "*We shall consider how concepts fit together to form conceptual structures, called schemas, and examine some of the results which follow from the organization of our knowledge into these structures*."

Skemp discusses the construction of concepts in general and the construction of mathematical concepts in particular. For Skemp, the *abstracting process* informs us about certain similarities between our experiences. *Classifying*, for Skemp, is the grouping of experiences based on these similarities (Hitt, 2002, p.243). He dedicated an entire chapter in his book to the concept of schema.

> "*The general psychological term for a mental structure is a schema. The term includes not only the complex conceptual structures of mathematics, but relatively simple structures which coordinate sensory-motor activity. […] A schema has two main functions. It integrates existing knowledge, and it is a mental tool for the acquisition of new knowledge*" (Skemp, 1971, p.39).

Skemp conducted research on students' comprehension and differentiated between *relational and instrumental approaches* to mathematics. According to Skemp (1978)

> […] *learning relational mathematics consists of building up a conceptual structure(schema) from which its possessor can (in principle) produce an unlimited number of plans for getting from any starting point within his schema to any finishing*

> *point. […] Building up a schema within a given area of knowledge becomes an intrinsically satisfying goal in itself. […] But a schema is never complete. As our schémas enlarge, so our awareness of possibilities is there by enlarged*. (p.14)

Therefore, a child understands a mathematical concept or a mathematical object only if it can be integrated into a schema that pre-exists in their mind.

2.0 THE CONCEPT OF SCHEMA ACCORDING TO PIAGET'S THEORY

In Piaget's theory (e.g., 1970, 1971), learning occurs when a child "matches" new experiences with the cognitive schemas they already possess. Anything repeatable and generalizable in an action, according to Piaget, is referred to as a *schema*. Therefore, a student is able to understand mathematical situations, is capable in solving the problems presented, and demonstrates cognitive growth. A schema on its own is not considered a logical structure; schemas coordinate with each other in the mind of the individual. Moreover, a schema can contain *subschemas*, according to Piaget. The categorization of schemas into specific categories and their interrelation forms *concepts* (Cooley, Trigueros & Baker, 2007, p. 372). *Cognitive schemas* represent the way a student (or a subject in general) interprets or logically organizes the surrounding world. This happens through two processes: (a) *assimilation*, where the subject incorporates information received from the external world into their existing cognitive structures, and (b) *accommodation*, where the cognitive structures are adjusted and modified to "fit" the new information.

> "*For Piaget, the essential building block for cognition is the scheme. A scheme is an organized pattern of action or thought. It is a broad concept and can refer to organized patterns of physical action (such as an infant reaching to grasp an object), or mental action (such as a high school student thinking about how to solve an algebra problem). As children interact with the environment, individual schemes become modified, combined, and reorganized to form more complex cognitive structures*" (Littlefield-Cook, & Cook, 2005, chapter 5, p.6).

Piaget and Garcia (1983/1989) consider that a schema develops in three stages [intra, inter, and trans] and summarize the process as follows:

> *The intra phase leads to the discovery of a set of properties in objects and events finding only local and particular explanations. The 'reasons' to be established can thus be found only in the relations between objects, which means that they can be found only in 'transformations.' These, by their nature, are characteristic of the inter level. Once discovered, these transformations require the establishment of relations between each other, which leads to the construction of 'structures,' characteristic of the trans level*. (pp. 273–274) (cited in Tsou, 2006).

In other words, in the first stage, subjects (and therefore students) focus on distinct areas of knowledge without having constructed any relationship between them. In the second stage, students begin to construct relationships between parts of mathematical knowledge. In the concluding phase, children, building upon ideas that has already been established in the earlier stage, construct the connections between different areas of knowledge. Piaget introduced the concepts of *empirical abstraction*, *pseudo–empirical abstraction* and *reflective abstraction* "*to* *describe the construction of logico–mathematical structures by an individual during the course of cognitive development*" (Dubinsky, 1991a, p.95).

- "*Empirical abstraction*: a subject (e.g., a student) proceeds to this kind of abstraction after the observable experience with a few objects through which the subject understands that these objects have a common property or in the words of Dubinsky "*the subject observes a number of objects and abstracts a common property*" (p.98) Empirical abstraction derives knowledge from the properties of objects (Beth & Piaget, 1966, pp.188–189).
- *Pseudo–empirical abstraction*: a subject (e.g., a student) proceeds to this kind of abstraction after the experience with actions performed on the objects (p.98). Pseudo–empirical abstraction "*is intermediate between empirical and reflective abstraction and teases out properties that the actions of the subject have introduced into objects*" (Piaget, 1985, pp.18–19).
- *Reflective abstraction*, "is completely internal" (p.97). Reflective abstraction is drawn from what Piaget (1980, pp. 89–97) called *the general coordinations of actions* and, as such, its source is the subject and it is completely internal.

The idea of reflective abstraction relates to actions and operations, functioning as a method of reflection or revision that enhances comprehension from a lower to a higher level. Additionally, it builds upon existing schemas, enabling a reorganization of knowledge at an advanced level, informed by the individual's prior experiences. (Beth & Piaget, 1966, p.88-89 as cited in Cooley et al., 2007, p. 374). According to Piaget, "*The development of cognitive structures is due to reflective abstraction*" (Piaget, 1985, p. 143)". Reflective abstraction is the construction of mental objects and of mental actions on these objects.

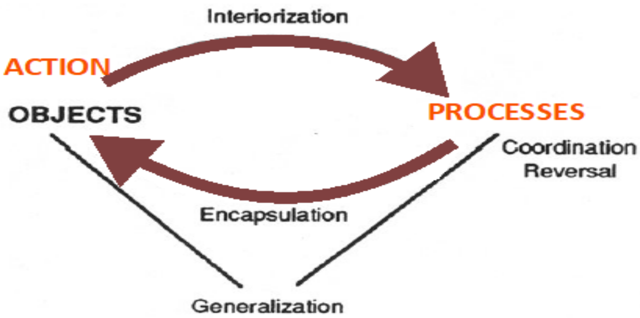

Figure 1. Schemas and their construction (Dubinsky, 1991a, p. 106, adapted)

Piaget found that the development of children's logical thinking could be described in terms of five sub-operations or forms of construction in reflective abstraction: *interiorization, coordination, encapsulation, generalization, and reversal* (Dubinsky, 1991a, p. 103):

- *Interiorization* "is the translation of a succession of material actions into a system of interiorized operations" (Beth & Piaget, 1966, p. 206, cited in Dubinsky, 1991a, p.100).
- *Coordination* "of successive displacements can form a continuous whole" (Piaget, 1980, p. 90, cited in Dubinsky, 1991a, p.100)

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- *Encapsulation* “of actions or operations become thematized objects of thought or assimilation” (Piaget, 1985, p. 49, cited in Dubinsky, 1991a, p.100).
- *Generalization* “is the passage from “*some”to“all”, from the specific to the general* (Piaget & Garcia, 1983, p. 299, cited in Dubinsky, 1991a, p.97). Generalization is a productive process that leads to new syntheses, during which specific laws gain new significance. Therefore, a subject learns to apply an existing schema to a broader set of phenomena (Piaget & Garcia, 1983, p. 299, as cited in Dubinsky, 1991a, p. 97).

Vosniadou (2006) states that in traditional mathematics teaching, the knowledge construction process typically depends on what is known as *empirical or horizontal generalization.* (e.g., Dubinsky, 1991a, b; Piaget, 1978). Generalizations are not based on the construction of an abstract theoretical model, but on the similarities between problems. Through *horizontal generalization*, students can solve problems commonly found in textbooks, but may struggle to develop a thorough understanding of concepts due to their limited mental models, which are based on concrete mathematical knowledge (/entities). If our aim is to delve into deeper and more abstract levels of mathematical knowledge, a different approach to knowledge construction is required.

For Dubinsky (1991a, b), schemas are structural organizations of actions, processes, and objects. Dubinsky (1991a, b), Asiala et al. (1996) introduced the *Action-Process-Object-Schema* (APOS) theory, a constructivist theory based on Piaget's theory. Dubinsky (1991a, b), Asiala et al. (1996), and Cottrill et al. (1996) view the concept of schema from the perspective of APOS theory: if we apply repeated actions to objects, these actions become internalized as processes, which are then encapsulated as mental objects, subsequently creating cognitive structures or schemas. For a subject to construct an "object," they must reflect on their actions. Schemas help us interpret and store new experiences guiding us in deciding which aspects require special attention, aiding the decoding process (Lieberman, 2004, p.411). Teachers can thus assist their students by paying attention to how they use their preexisting knowledge through the answers they give and the way they communicate mathematical ideas. This approach enables students to verbally articulate, during classroom teaching, the specific and abstract situations they encounter in problems. According to Lakoff & Nùñez (2000), schemas are conceptual mechanisms in mathematics -a mechanism by which the abstract becomes comprehensible- or *mental representations*, or *conceptual metaphors*, as well as conceptual combinations of these. The "schema" concept has been used in mathematics education to explain how students build, rebuild, and use their knowledge when they solve problems (Steffe, 1983).

3.0 VERGNAUD’S CONCEPT OF SCHEMA

The concept of schema also holds a central position in Vergnaud's theory of *conceptual fields* (e.g., 1988, 1990, 1996, 1998, 2009). The theory of conceptual fields is the "reference theory" for many authors concerning systems of representation (Hoyles & Noss, 1996; Kaput, 1992). Zazkis & Liljedahl (2002) further argue that

> “*The development of the theory of conceptual fields was motivated by the need to establish connections among explicit mathematical concepts, relations, and theorems,*

and between students' (at times implicit) dynamic conceptions and competencies related to these mathematical concepts, relations, and theorems." (p. 95).

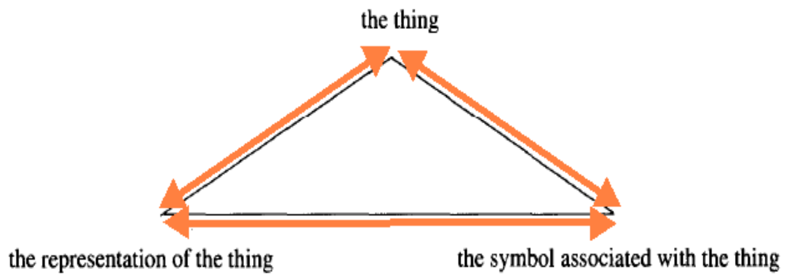

Figure 2. The metaphor of the Aristotle's Triangle for the meaning of representation (Vergnaud, 1998, p. 168) (adapted)

As previously discussed, schemas "coordinate and organize" the observable patterns of a subject's activity with their mental representations. In the words of Vergnaud (1998, p. 172), the concept of schema "*is more general and relates to different kinds of activities*." Examples of schemas for Vergnaud include telling a story, constructing a political speech, speaking a foreign language fluently with certain specific mistakes and accents, and engaging in dialogue with a group of people. These are what he calls *verbal schemes or social schemes* (Vergnaud, 1998, p. 172). These categories of schemas play a crucial role in the field of education, especially within the context of mathematics education. This is due to the fact that when a student encounters a novel mathematical scenario in class, they are able to recall a set of schemas that they have previously developed. These situations result in the creation of novel mathematical schemas or the enhancement of existing ones. For instance, when some students collaborate to solve a problem, they may develop social and verbal schemas, as well as schemas related to mathematical concepts.

The concept of schema is the most important concept in cognitive psychology (Vergnaud, 1998, p. 172). A schema, for Vergnaud, is the invariant organization of behavior for a specific category of situations. According to Vergnaud, a mental schema consists of four basic components (1998, p.173): (a) *goals and anticipations* to achieve potential objective goals, (b) *rules-of-action* (: the rules that generalize the sequence of actions to be taken, typically in "if... then" forms that contain the operational invariants used by the subject in practice), (c) *operational invariants*, which include the concepts of *theorem-in-action and concept-in-action* that form the cognitive content of schemas, and (d) *inference possibilities* (:the outcomes determined based on the information available to the subject). As Vergnaud (1996) points out, "*by the term concept-in-action, I mean an implicit concept, and by the term theorem-in-action, a proposition that can be considered true. There is no theorem-in-action without a concept-in-action and vice versa, as a theorem makes no sense without an implied concept*" (p. 225). A theorem-in-action (Vergnaud, 1996, p. 225) is thus a proposition that the subject considers true for a category of variable situations or in relation to the real world. According to Vergnaud, schemas give shape to all possible types of behavior, including gestures, the individual's mental activities, and their linguistic behavior (Stipcich, Moreira & Sahelices, 2007).

Vergnaud (1998, p.168) provides examples of theorems-in-action, which emerge during the solution of the following problem by students: *A train is running at a constant and high speed. It takes 16 minutes to go from Axis to Berlof. The distance between Axis and Berlof is 40 km. From Berlof to Cadillac, it takes 36 minutes. What is the distance between Berlof and Cadillac*? In other words: It takes 16 minutes for the train to travel from city X to city Y, which are 40 km apart, and 36 minutes to travel from city Y to city Z. The distance between city Y and city Z is t. Vergnaud outlines schematically the operations two students perform to arrive at the solution: "[1st student: 40 × 2 = 80, 80 + 10 = 90], [2nd student: 40 ÷ 16 = 2.5, 36 × 2.5 = 90]."Later in his article, he leads into the following analysis in order to understand the way of thinking of the two students and how they develop theorems and concepts-in-action (Figure 3a, b, c, d).

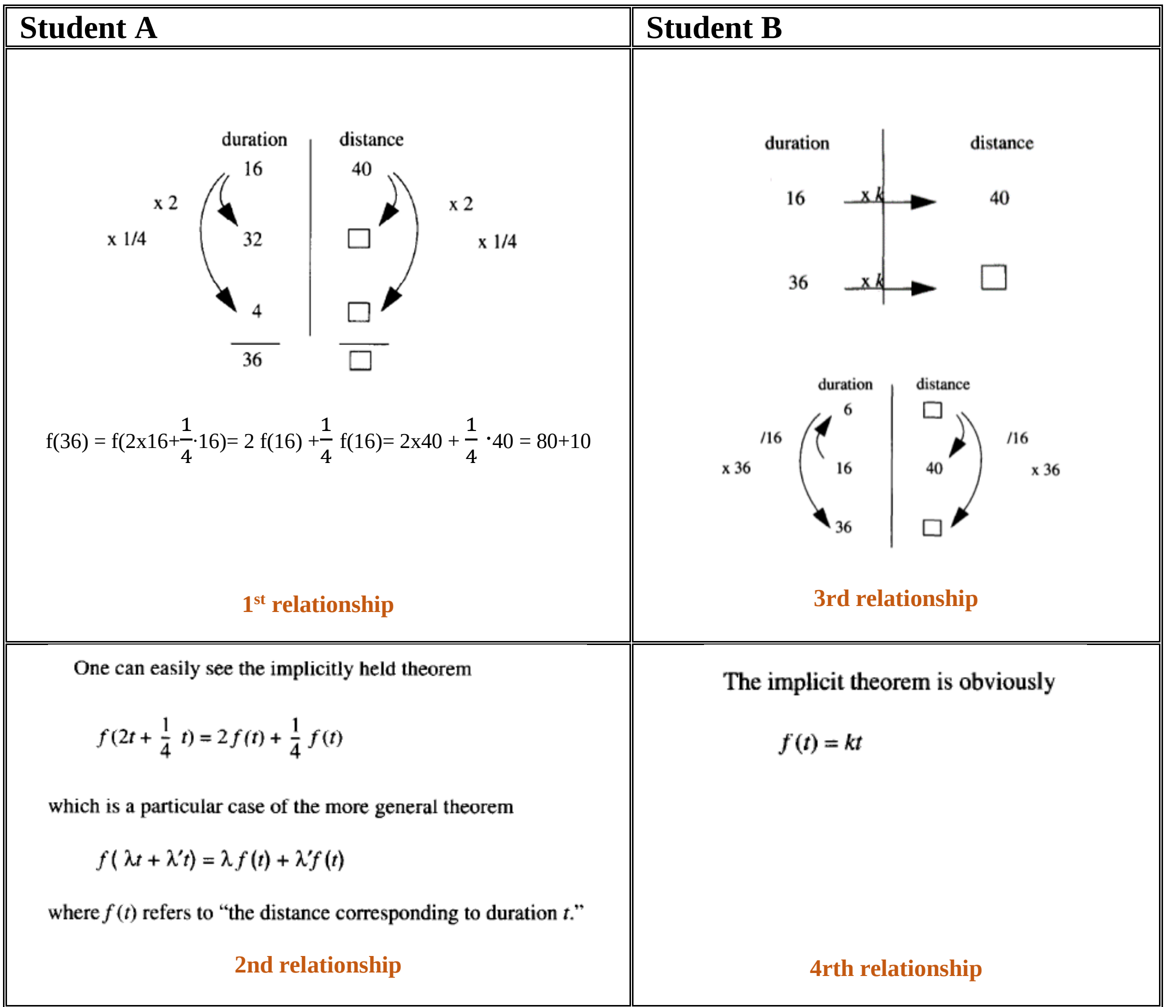

Figure 3 a, b, c, d. Theorems -in-action and concepts-in-action evoked during the problem-solving process (Vergnaud, 1998, pp.168-170)

According to Vergnaud, the students arrive at a theorem they did not know beforehand, but which arose during their actions on the problem. For example, student A has solved the problem in the way we already mentioned, while the first relationship is the strict relationship with which he would solve the problem (if he had the knowledge), the generalization of the

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procedure leads to the second relationship. For Vergnaud (1998, p. 167), the concept of representation is a significant object of study for two main reasons: Firstly, we perceive representations as internal images, gestures, and words. Secondly, the words and symbols we use to communicate do not directly refer to reality but to represented entities, such as objects, properties, relationships, processes, actions and constructs. As a result, mathematical concepts are rooted in actions upon the physical and social world and their representations. Therefore, representation *is a dynamic process rather than a static one* (Vergnaud, 1998, p. 167). Consequently, *a concept* is a set of situations that involve a set of different operational invariants and their properties, which can be expressed through various symbolic and verbal representations (Vergnaud, 1998, p. 177). In the context of the discussion regarding the process of rediscovery-reinvention of concepts through representational systems, it is necessary to answer certain questions, such as (Vergnaud, 1990): *what is the nature and function of a new concept, a new process, a new type of reasoning, a new representation?* Vergnaud (1998) highlights a significant aspect regarding the reference to reality, which can represent two distinct concepts: reference to objects and reference to situations. Furthermore, the *signified* pertains to the cognitive processes that take place, such as prediction and conclusion drawing, and relates to the individual's internal representation of the world. In this context, constants are recognized, conclusions are drawn, and actions and predictions are formulated. Additionally, the signifier encompasses various symbolic systems, including shapes and diagrams that facilitate the explanation and interpretation of the situation from a mathematical viewpoint. Consequently, according to the theory of conceptual fields, knowledge can manifest in two ways (Zazkis, & Liljedahl, 2002, p.97): Recognizing the differences between two seemingly similar scenarios allows for the adoption of unique theorems-in-action. Furthermore, by uncovering the common framework that connects two previously regarded distinct sets of situations, one can modify the two theorems-in-action to develop a more comprehensive theorem-in-action applicable to both sets of circumstances.

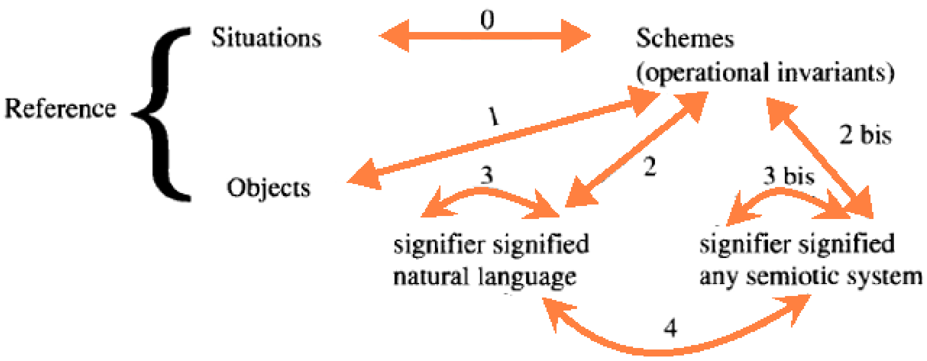

Figure 4. The notion of representation (Vergnaud, 1998, p.177) (adapted)

Hence, it could be inferred that a student's schema integrates theorems-in-action that give rise to concepts-in-actions. When students encounter a novel situation, they draw upon the knowledge they have acquired from previous, less complex experiences to adapt to the new context (Vergnaud, 1988, p.141). This process is related to Piaget's processes of accommodation and assimilation. The development of conceptual frameworks in mathematics relies on the activities carried out by students both individually and as part of a group or class, as well as their engagement with other members of the school community [especially with the educator and their classmates]. Furthermore, Mariotti (2002) considers that the constructivist result produced through reflective abstraction in problem-solving activity is consistent with Vergnaud's (1990) theory of conceptual fields.

4.0 FROM THE THEORIES OF PIAGET AND VERNAUD TO THE SOCIAL - COGNITIVE PERSPECTIVES

According to Piaget's theory, the interaction between individuals and their environment plays a crucial role in the learning process, as it is shaped by their pre-existing knowledge. The construction of mathematical concepts involves the utilization of cognitive functions to form a "schema" for the given concept. These schemas are developed in students minds through scenarios that involve problem-solving, where they engage with computational educational tools within the classroom setting. In a social learning environment, children enhance or adjust their schemas through interactions with their peers, educators, and both static and dynamic tools and artifacts. The sociocultural theories (e.g., Vygotsky, 1978) report and investigate the cognitive processes engaged in a social and cultural setting. Vygotsky's (1978) viewpoints within *Activity Theory* offer an opportunity for a deeper investigation and analysis of the schema concept, along with the complex network of connections that are developed during the teaching and learning process in an environment facilitated by instruments and tools.

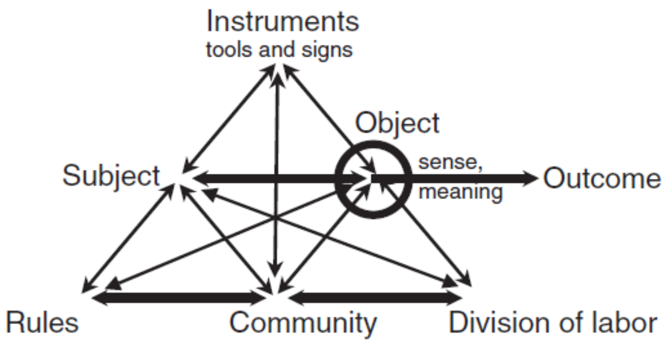

Figure 5. General model of an activity system (Engeström, 1987, p. 78)

Leontiev (1981) originally developed Activity Theory, which is now widely utilized across various disciplines such as mathematics education and human-computer interaction. It offers a suitable framework for analysing technological innovations as it is part of the broader process of sociocultural evolution, where artifacts mediate human activities (Bellamy, 1996, cited in Nardi, 1996). The questions posed relate to the complexity of learning, regarding culture and knowledge acquisition. Activity Theory includes the following interactive components: subjects, tools or artifacts that mediate, the object, etc., interpreted as follows (e.g., Jonassen et al., 1999; Engeström, 2001):

- *Subject:* It could be an individual or group engaged in the activity.
- *Object:* It is the physical or mental product that is transformed, that mobilizes the entire activity, giving a specific direction.

- *Tools or artifacts:* Anything used in the process. Their use shapes how people act and think. Tools reshape activity and vice versa; activity is shaped by tools. Kuutti (1996) considers tools can be physical tools or tools of thought.
- *Activity:* It consists of goal-directed actions used to complete the result.
- *Rules:* They cover explicit or implied norms and relationships within a community.
- *Division of labor* refers to the organization of the community to produce results.

Kuutti (1991) discusses Activity Theory as a philosophical and interdisciplinary theory that focuses on the examination of human practices, including teaching and learning. These practices are facilitated by artifacts, with students engaging in both individual and collaborative work within a social classroom setting. This environment allows for the simultaneous integration of individual and social dimensions of knowledge. Cerulli, Pedemonte, and Robotti (2005, p.1392) assert that the core concept of activity theory is "activity". Educational activities play a crucial role in enhancing students' comprehension. Examples include attending presentations (such as videos or interactive whiteboards), exploring geometric concepts in a dynamic geometry software environment, or studying from a textbook. The collaborative work of Vygotsky, Luria, and Leontiev in activity theory, according to Hoyles (2003), demonstrates that the actions of a student (individual actions) or actions resulting from interaction with the community (social actions) during problem-solving depend on the artifacts with which subjects interact. Engeström's (2001) model explores human activities in a comprehensive manner, while also shedding light on the intricate web of relationships that characterize teaching and learning processes (Nardi, 1996). As per this model, an activity may undergo development, leading to a shift in its objective and subsequently altering its structure. Put simply, as the activity advances, the actions of the student, teacher, or external factors can trigger modifications in the relationships that define the activity.

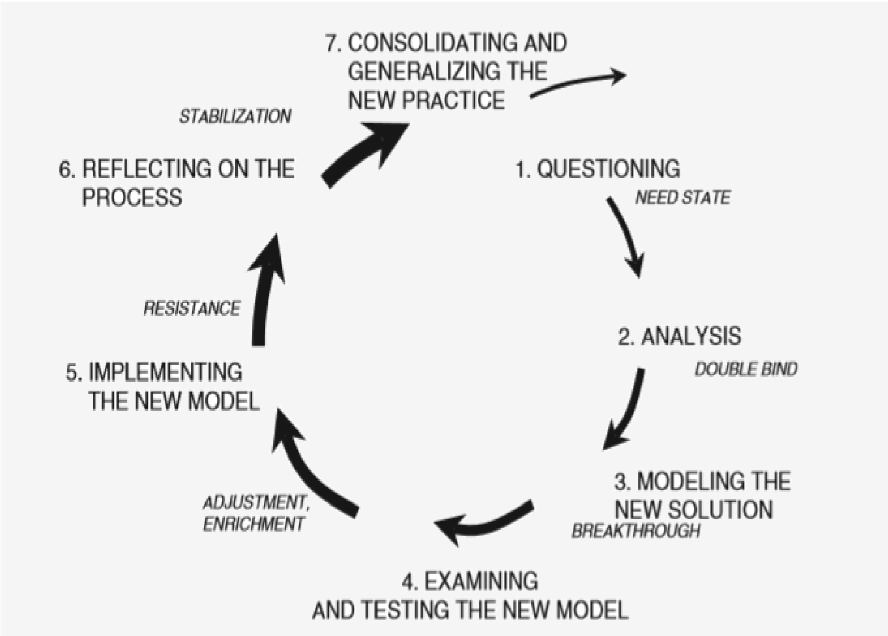

Figure 6. Sequence of learning actions in an expansive learning cycle (Engeström, 2010, p. 80)

On the other hand, *expansive learning* is “activity is an activity-producing activity” (Engeström, 1987, p. 125). According to Engeström (2010), integrating Activity Theory and expansive learning into learning theories requires consideration of three dimensions:

1. *Is learning primarily a process that transmits and preserves culture or a process that transforms and creates culture?*
2. *Is learning primarily a process of vertical improvement along some uniform scales of competence or horizontal movement, exchange and hybridization between different cultural contexts and standards of competence?*
3. *Is learning primarily a process of acquiring and process that leads to the formation of theoretical knowledge and concepts?* (Engeström, 2010, p. 74)

Introducing an artifact into a situation can resolve existing problems, but it also changes the nature of goals /objectives, necessitating new tools for their resolution (Nardi, 1996). Different tools can be seen as "external symbols" that represent a particular mathematical concept (e.g. the compass as a tool, an artifact associated with the concept of a circle). Therefore, as instruments of *semiotic mediation* (Mariotti, 2002), they act as mediators between the individual and the objectives. Teachers play a crucial role in the educational setting by not only organizing the learning environment but also selecting the tools and artifacts necessary for semiotic mediation. Student progress in the classroom is a collaborative effort, involving interactions with peers and guidance from the teacher (Cerulli, 2004; Cerulli & Mariotti, 2003). The teacher facilitates the use of tools to enhance learning and communication strategies aimed at grasping specific concepts. According to Mariotti (2000), Vygotsky highlights that humans create and use tools and signs, which serve as a reflection of cultural heritage. Vygotsky distinctly differentiates between signs and tools, even though both are categorized as mediators. (1978, p. 53). The difference between the two elements is based on *the different ways they orient human behavior* (Vygotsky, 1978, p. 54):

1. *Tools* serve as a means for humans to interact with and control the external world, facilitating the organization and mastery of nature.
2. *Signs, or psychological tools,* serve as a means for individuals to internally organize themselves, directing their internal activities towards personal organization.

According to Vygotsky, the two processes are closely linked, just as reshaping nature by humans, reshapes human nature itself (Vygotsky, 1978, p. 55).

> “*The person, using the power of things or stimuli, controls his own behavior through them, grouping them, putting them together, sorting them. In other words, the great uniqueness of the will consists of man having no power over his own behavior other than the power that things have over his behavior. But man, subjects to himself the power of things over behavior, makes them serve his own purposes and controls that power as he wants. He changes the environment with the external activity and in this way affects his own behavior, subjecting it to his own authority”* (Vygotsky, 1997, p. 212, cited in Engeström, 2010, p.77).

Thus, when a student utilizes a tool to accomplish a goal in an activity, learning takes place as a result of constructing knowledge through the nature of the activity itself. In this context, the mathematics teacher plays a crucial role in shaping discussions and in organizing and selecting mediation tools, through which mathematical concepts are introduced or explored. The outcomes of learning, as a result of interaction with activities, may not align with the teacher's educational goals (Cerulli, Pedemonte & Robotti, 2005).

4.1 The concept of schema in an instrumental genesis process

An artifact undergoes a metamorphosis into a tool through the concept of "*instrumental genesis"* (Verillon & Rabardel, 1995), which is influenced by the user and the desired outcome.

Instrumental genesis has two components (Guin, & Trouche, 2002, p.205): (a) *an instrumentalization component*, related to the artifact; (b) - *an instrumentation component* related to the organization of the subject's behaviour. The process of instrumentalization can involve various phases (Guin & Trouche, 2002, p.205): *the discovery phase*, the *personalization phase*, and the evolution or transformation of the artifact, often deviating from its original intended purpose. As per instrumentation process: "*this process is relative to the emergence and evolution of schemes of a subject for the execution of a given task*" (Guin & Trouche, 2002, p.205). The process of instrumentation involves the differentiation of artifacts themselves. According to Guin & Trouche (2002), this process is linked to the development of a subject's strategies for reaching a goal, as outlined in Vergnaud's theory (1996) on schema, which is an invariant structure consisting of theorems-in-action and concept-in-action. Throughout the instrumental genesis process, both phases coexist and interact, enabling the subject to actively create what Rabardel (1995) refers to as "utilization schemes" or "usage schemes" of the tool. These cognitive schemes help organize activity through the tool to achieve a specific objective. In Rabardel's publication "People and Technology: a cognitive approach to contemporary instruments" (Rabardel, 1995, p.84), are identified various types of utilization schemes for a tool:

- *Usage schemes related to "secondary tasks":* schemes oriented towards managing the tool.
- *Instrumented action schemes* which consist of wholes deriving their meaning from the global action which aims at operating *transformations on the object of activity*: schemes oriented towards executing a specific goal.
- *Instrument-mediated collective activity schemes* which concern the specification of the types of action or activity, of the types of acceptable results etc. *when the group shares a same instrument or works with a same class of instruments*: schemes concerning the coordination of actions of individual persons as a contribution to the success of common goals.

Rabardel (1995) further specifies two principles related to the production of utilization schemes by the subject:

- *The "principle of economy*", which is an economic principle sought by the user. Consequently, when two distinct objectives are presented to the user, particularly designed to complement one another, and when the subject interrupts while performing the initial goal in order to complete the other, they tend to use the tool they used for the initial goal to achieve the new goal.

- *The "search for efficiency*", where if the subject considers that the proposed tool will not be the most efficient regarding the goals they want to achieve, they tend either to choose another tool or to use the proposed tool in a way that the designers of the tool did not foresee (informal use or "*catachrèse*," according to Rabardel) (p.96).

According to Rabardel, the use of tools can have both individual and social dimensions. The individual dimension refers to how each person uses a tool, while the social dimension involves the development of schemas during a process in which the subject participates. Designers of artifacts, such as software or software activities, aim to design the tool in a way that contributes to the formation of these types of schemes. This is especially important for tools designed to teach mathematical concepts, as they can contribute to the formation of collective action schemes among students.

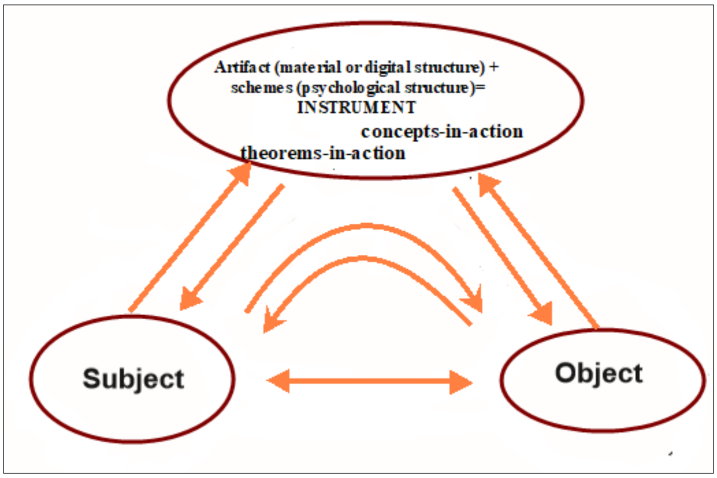

Figure 7. The mediating instrument (Beguin & Rabardel, 2000, p. 179) (adapted)

Trouche (2004, p.292) discusses examples concerning instrumented action schemes and the formation of theorems -in -action during interaction with tools or artifacts used by students:

> “*The student in this example may explain, for instance, that considering the function graphic allows him to conclude the following: ‘‘if the function increases with great speed, it is okay. On the other hand, if the function starts to decrease or oscillate then it is not good.’’ Consequently, one can put forward the hypothesis that the student’s scheme integrates theorems-in-action of the following type ‘‘if the function increases very strongly, then the limit of f is infinite’’, ‘‘if the limit of f is infinite, then f is necessarily increasing’’...*"

Trouche (2004, p.154) posits that during the initial stage, gestures are guided by operative invariants as they go through the processes of exploration, conclusion, and justification. Trouche views the correlation between the concepts of "scheme" and "gestures" as a means of linking internal and external phenomena (personal communication via email, October 2007).

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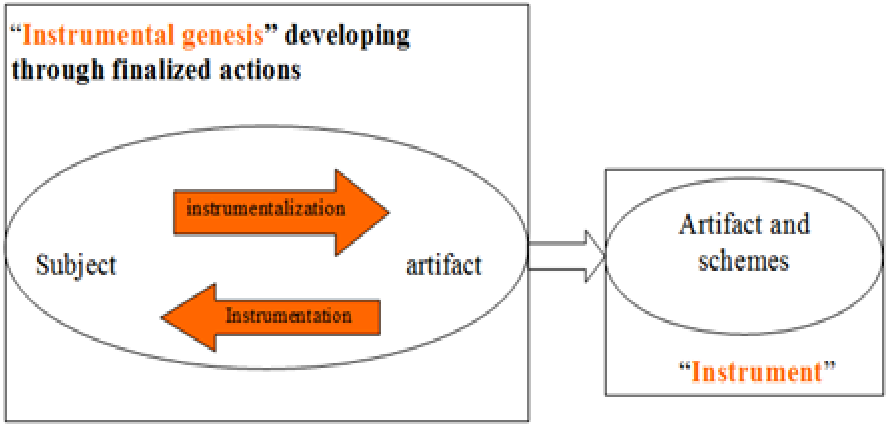

Figure 8. The framework of the instrumental approach

From Trouche's point of view, "instrumental geneses are individual processes, developing inside and outside classrooms, but including of course social aspects" (Figure 8) (personal e-mail correspondence with Professor Trouche on April 2-4, 2008). Trouche supports that "*an artefact is transformed thus through instrumental geneses, oriented by finalized actions, assisted by instrumental orchestrations, into an instrument*".

According to Trouche (2003), the psychological component of the instrumental approach is explained through the Piagetian concept of schema, as "*the structure or organization of actions as they are transferred or generalized to similar or analogous situations*" (Piaget & Inhelder, 1969, p.4). Rabardel & Beguin (2005, p.439) refer to the concept of schema as follows: "*What is a schema? According to Piaget, schemas are behavioral reorganizers. An action scheme is a structured set of active characteristics that can be generalized, which can, for example, make it possible to repeat the same action or apply this action to new situations (Piaget & Beth, 1961). A schema consists of a set of mutually dependent components that cannot function separately*."

In the words of Rabardel (2002, p.85) "Utilization schemes are multi-functional as they carry out: (a) epistemic functions focused on understanding situations; (b) *pragmatic functions* focused on transforming the situation and obtaining results; (c) *heuristic functions* that orient and control the activity". Hence, the genesis of instruments is a multifaceted procedure that links the capabilities and limitations of the tool with the actions of the individuals. Trouche also presents additional concepts such as *techniques and instrumented techniques* (Trouche, 2005a, b). Techniques refer to a series of movements executed while utilizing a technological tool to accomplish an objective (e.g., resolving an issue). On the other hand, instrumented techniques are methods that externalize the patterns of instrumental behaviour of the individual and typically require multiple tools to achieve a goal. Beguin (2006) emphasizes the importance of placing the instrument at the core of the educational tool design process. By developing an "*instrumental proposal*" he suggests that designers should aim to shape instrumented action schemes using the tool. This focus on instrumental genesis process can aid in creating the "*instrument*", which is the psychological tool that includes its usage and the concepts that arise from it. Defining the design process as a dialogical one emphasizes that an artifact serves as a link between different stakeholders, each with their own unique viewpoints.

Essentially, the concept is that a practical proposal materializes in the artifact as a reflection of the user's actions. It is important to recognize that in designing an artifact, designers envision a purpose that guides the user's actions. However, this purpose is essentially a proposal crafted by the designers, and there will be a corresponding "response" from users during the instrumental genesis.

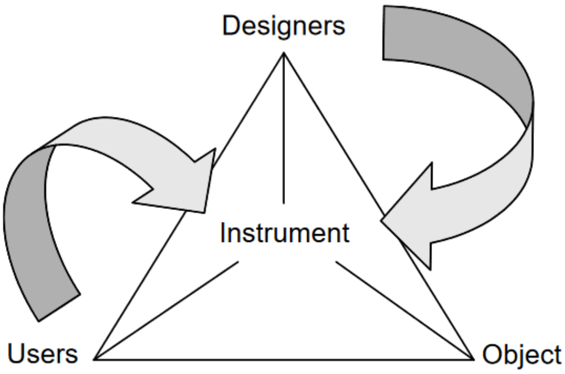

Figure 9. The ‘‘instrumental proposal’’ and the ‘‘instrumental genesis’’ in dialogue (Beguin, 2006, p.15)

5.0 FROM SCHEMES TO SOCIAL SCHEMES

As previously stated, social schemes are formed through social interaction and are influenced by the use of tools and artifacts within a specific environment. According to Rabardel & Samurçay (2001), social schemes are the mental constructs that individuals adopt, even when they are part of or influenced by educational processes. As they state "*We have moved beyond this limitation by giving utilization schemes the characteristics of social schemes: they are elaborated and shared in communities of practice and may give rise to an appropriation by subjects, or even result from explicit training processes*" (p.20). This definition by Rabardel & Samurçay (2001) bridges the gap between Piaget's genetic epistemology and Vygotsky's concept of semiotic mediation:

> "[…]*we were able to bring together theoretical, Piagetian and post-Piagetian perspectives on the role of action and activity in the genesis of knowledge and conceptualization, with the central role given to the mediation by cultural artifacts in activity theories. The Piagetian concept of the scheme as a structural invariant of the action allows us to identify the replicability of the action and, to a certain extent, the activity in its range of contexts and situations*" (Rabardel & Samurçay, 2001, p.19).

Trouche (2004) states "*a scheme, according to Piaget, is a means for assimilating a situation encountered by a subject, but at the same time a scheme, as noted by Rabardel and Samurçay (ibid.), is the result of an 'assimilative' activity, where the environment and available tools play a significant role*." Trouche (2004, p.216) illustrates an orchestration process where a student takes on the role of managing a computer and is designated as the "sherpa-student." This student serves as a guide for the group, similar to a Sherpa in the Himalaya mountains who assists in carrying loads or preparing diplomats for conferences. The "sherpa-student" is viewed as a reference point by both the teacher and the class, acting as a guide, assistant, and mediator.

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The orchestration process consolidates the collaborative tool utilization as the "sherpa-student" works on their personal computer, enabling others to observe it (on their screens or through a projector). This facilitates the comparison of various tool utilization methods and offers the teacher insights into the types of tool-based activities that can be developed based on the student's Sherpa role. Additionally, the teacher guides, via the student's computer, without executing the tool-related tasks—actions carried out by the student. In this manner, the teacher takes on a role similar to that of an "orchestra conductor". Trouche introduced the notion of instrumental orchestration as follows (2004, p. 296):

> I introduce the term *instrumental orchestrations* to point out the necessity (for a given institution: a teacher in her/his class for example) of *an external guidance* of students' instrumental genesis. This necessity is rarely taken into account: one can find in textbooks or papers relating CAS experiments comments on material components (calculators or computers, kinds of software, overhead projectors, directions for use) and on didactical components (exposition of the mathematical subject and of different stages of treatment), but seldom information on the environmental organization, i.e. on the organization of the students' and/or teachers' work space and time.
>
> Trouche (2004)

This passage highlights the intricate relationship between cognitive development, tool mediation, and the practical organization of educational settings as discussed by Trouche and contextualized by Rabardel and Samurçay. The instrumental orchestration is defined by four components (Guin & Trouche, 2002): (a) a set of individuals, (b) a set of objectives associated with achieving a type of goal or organizing a work environment, (c) a didactic configuration, which is a general structure of the action plan and (d) a set of exploitation of this configuration.

According to Noss & Hoyles (1996, p.105), the importance of orchestrations becomes evident when studying mathematics, as it serves as a tool for establishing connections among various problem categories, mathematical entities, and relationships between these entities. The following excerpt (Patsiomitou, 2008, p. 379) outlines my role as the researcher in the application of instrumental orchestration throughout the experimental phase of my Ph.D. thesis study:

"*All the students were free to click on and use the software tools to take an active part in the activity, and their contribution was the subject of discussion among the students. In that way, the mistakes made and the approach taken by each student could be seen by all, and they could all actively participate in the procedure; as a result, the class was more like a playgroup than a mathematics class working to strictly find proof. As they took part in the process and contributed to the proof by answering the researcher's (S.P.) questions, they added to it by explaining their conjecture, which was connected with the appearance of an additional constructive procedure that would appear. During the classroom experimental session, the researcher strove to activate those students that had not participated in solving the lost treasure problem during the pair-work phase, encouraging them to state and share their views. For example, students M7 to M10--the students who had the highest RVR (Reflective Visual Reaction) --had been involved in solving other problems, but had never done so in the multiple linking pages provided by the software. The process effectively stimulated cooperation as the team as a whole struggled to solve the problem in the best way they could. A presentation of the session's most important points, at which the pupils answered crucial questions, follows below. The session was approximately 25 minutes long*".

The students interacted with the mathematical concepts through the *Linking Visual Active Representations* (e.g., Patsiomitou, 2008, 2012a,b, 2019) that I designed in dynamic geometry software, as I encouraged everyone to participate in the process. The groups gradually investigated all the activities included in the *dynamic hypothetical learning path* that I designed and redesigned (Patsiomitou, 2012a,b) over a period of approximately four months, in order to examine all the emerging cases. The entire process resembled a dynamic path that, despite having predefined teaching and learning objectives, was partially altered by new situations that arose along the way. Through the dynamic learning path, a network of relationships between concepts was created.

During the instrumental orchestration, all students had the opportunity to take control of the mouse and interact with other students, expressing their opinions in interaction with the computer screen. Consequently, they created instrumental schemas throughout the process.

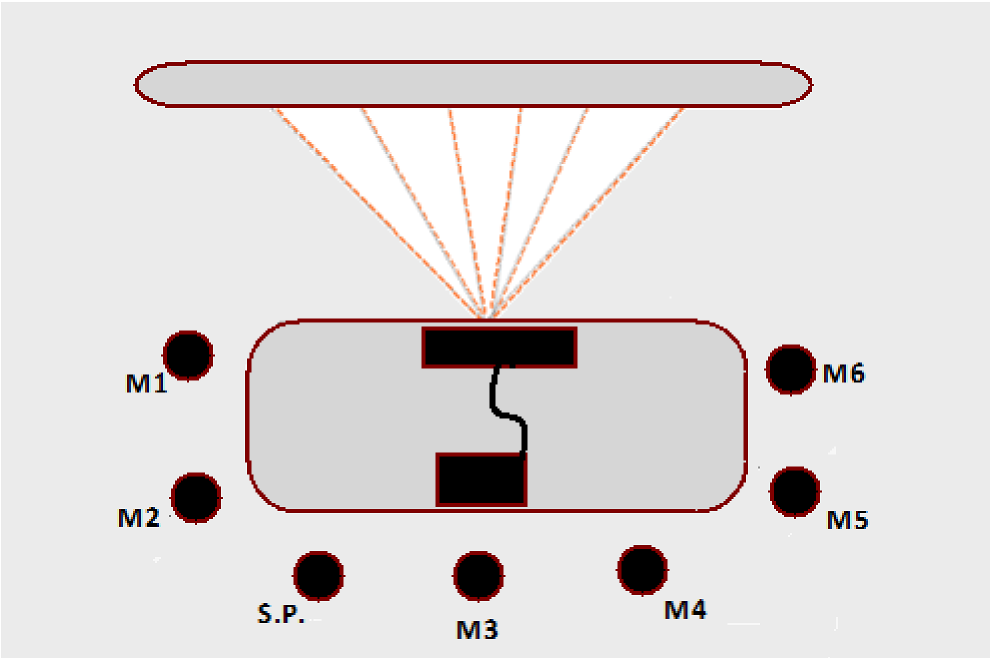

Figure 10. Dynamic instrumental orchestration process (Patsiomitou, 2008, p. 379)

In my studies (e.g. Patsiomitou, 2008, 2011, 2012, 2019, 2021 a, b) I analyzed, using instrumental decoding and pseudo-Toulmin modeling, several *conceptual and instrumental learning trajectories* for the teaching of, or the conducting of action research into Geometry, which employ digital dynamic means: in other words, the construction of instrumental learning trajectories within "Dynamic Euclidean Geometry".

Digital media and artificial intelligence have made a strong entry into the educational field, opening up new prospects for improving the quality of educational work and the evaluation of school units (e.g., Anastasiades, 2023). These tools, especially when combined with interactivity, provide an evolving framework that aligns well with the goals of LVAR in promoting deeper understanding and dynamic learning experiences

6.0 DISCUSSION

The process of learning and teaching Mathematics is perceived as both an individual endeavor and a socially constructed activity, with its framework and content shaped by the distinct objectives of education. In this context, it can be argued that cognitive tools, including the frameworks of mathematical concepts and the concepts themselves, are developed through a combination of individual and social interactions. Therefore, it is essential to develop digital environments that inspire students to build conceptual frameworks. This involves a variety of tasks and situations that promote students' understanding and application of abstract processes required for advancing to higher levels of cognition. Thus, the educational experience is not merely about delivering information; it also requires the active participation of students in comprehending and engaging with mathematical concepts through structured educational approaches that encourage reflective thinking and abstraction. Rabardel (2000) highlights the importance of the diverse artifacts and psychological tools we provide to students, which aid them in the complex process of instrumental genesis and the development of mental schemas. In summarizing, for Piaget, a schema is something that is repeatable and generalizable in action (Piaget, 1970, p.42). It is based on interactions with an emphasis on coordinating actions. In this sense, a schema is expressed through the use of symbols and tools, thus it is an instrumental scheme (Rivera, 2007). On the other hand, a tool is the result of cultural evolution and is created to serve specific purposes, thus it incorporates ideas. Tools always have social elements, being products of social experience. From this perspective, schemas have social components, and the genesis of tools has both individual and social aspects (e.g., Trouche, 2003, 2004). Figure 11 illustrates a representative timeline illustrating the concept of the schema as outlined in the present study. It traces the evolution of the notion of schema from the times of Aristotle and Plato's Academy through to the contributions of Piaget, Vergnaud, and Rabardel, highlighting its essential role in mathematics learning and the cognitive development of students.

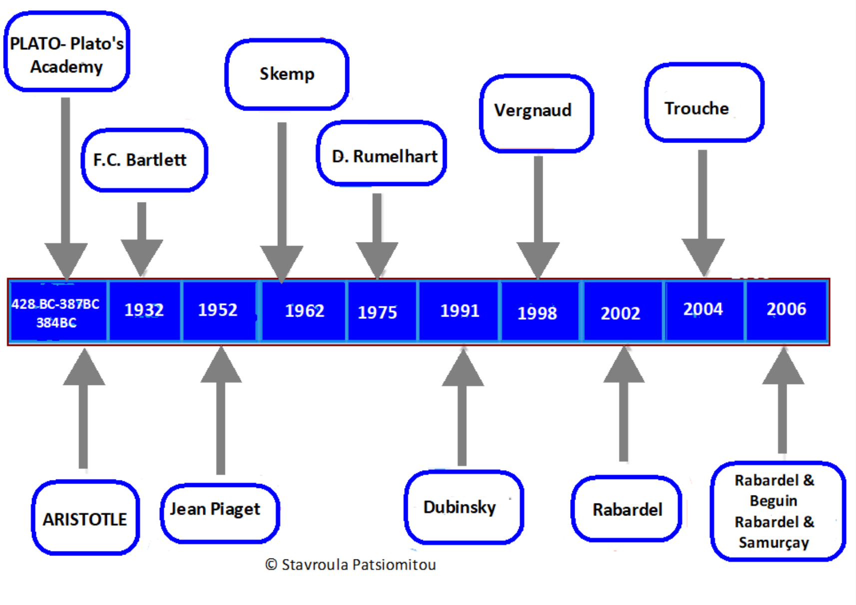

Figure 11. A representative timeline related to the concept of schema in the domains of Didactics and the Psychology of Mathematics, as discussed in this article

Lehtinen and Repo (1996) emphasize the significance of the representational tools employed by students, along with additional resources that facilitate social interaction, to foster the development of students' *reflective abstraction*. Additionally, the educator's guidance is vital, particularly through a structured sequence of activities that introduce concepts via various representations and suitable modelling within a computer environment (Figure 12).

Dynamic Hypothetical Learning Paths (DHLP), designed by educators utilizing Linking Visual Active Representations, offer a framework aimed at the development of both individual and social schemas for students (e.g., Patsiomitou, 2012, 2014). In this framework, despite the existence of predefined activities that utilize software methods and digital or AI tools, cognitive or instrumental obstacles (e.g., Patsiomitou, 2011) or other factors may necessitate a redesign, engaging with students in the process. Dynamic instrumental orchestrations, utilizing dynamic learning paths, are at the center of interest regarding the transformation of processes and concepts in teaching and learning, for the development of conceptual structures and abstract thinking processes in students.

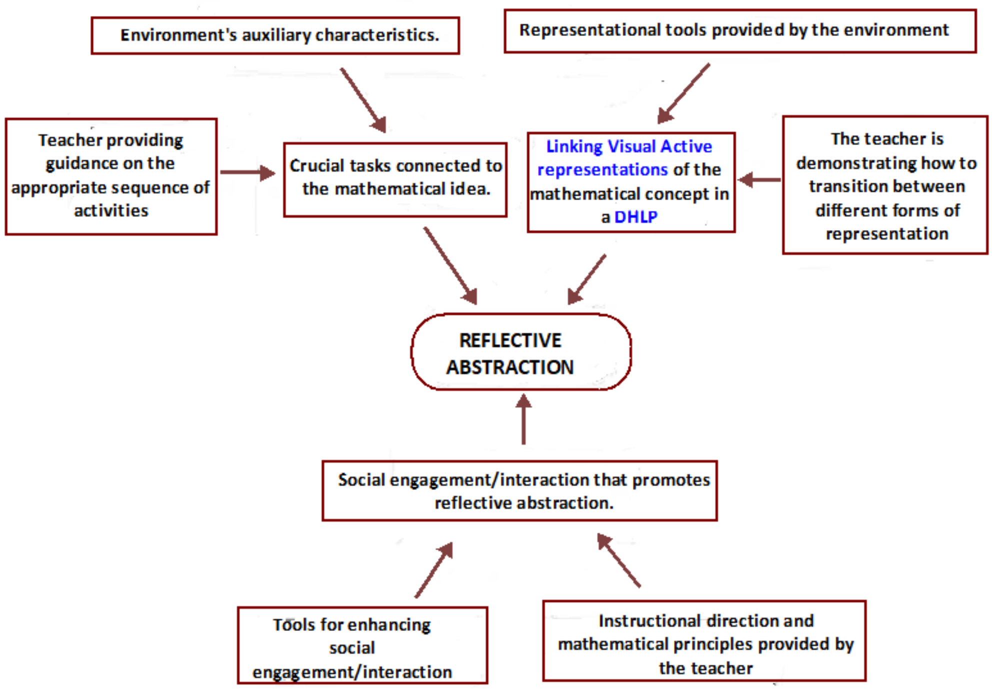

Figure 12. Presupposes for the development of reflective abstraction (Lehtinen & Repo, 1996, p. 113) (adapted)

What is the significance of digital tools and artificial intelligence in fostering reflective abstraction and, as a result, influencing cognitive and "instrumental" schemas?

In what ways do digital technologies, including artificial intelligence and specifically dynamic geometry software, assist in the creation of knowledge units that act as schemas for organizing and retrieving information? Additionally, how do these tools enhance the reuse and management of schemas in various contexts?

7.0 CONCLUDING REMARKS

In writing the epilogue, I thought I would have an opportunity to discuss dynamic geometry in education, as well the role of AI tools: where it is now, where I anticipate it being a few years from now, and what role the concepts I coined and introduced will play in that evolution (see also Patsiomitou, 2023, 2024). A point of reflection on the role dynamic geometry has played in education – and, at the same time, evidence on the impact it has on the thinking of students and users in general – can be got from Google and a scroll through the multitude of articles, dissertations and Ph.D. theses written about it around the world.

This evidence is overwhelming, as is the research that has been conducted into--and using--dynamic geometry software which describes its role in education in terms of the visualization and understanding of mathematical concepts, the development of students' levels of geometric thinking, and the development of proof/proving processes. In the realm of learning with technological tools, a teaching and learning scenario (Allen et al., 1995; Lejeune & Pernin, 2004; Trouche, 2004, 2005 a, b) outlines the roles, activities, and essential resources and tools required to accomplish each task.

Schools are always evolving. The integration of blended teaching methods with Artificial intelligence tools has profoundly changed educational practices in schools, influencing the development of various activities found in student textbooks and teaching resources. Dynamic geometry software and Artificial Intelligence tools play a crucial role in the instruction and comprehension of concepts in various scientific disciplines, spanning all educational stages from early childhood education to higher education.

Conversely, the activities of exploring, supervising, reflecting, analyzing, reorganizing, developing, launching, and implementing new ideas, theories, and terminology within the context of a comprehensive research project are essential for promoting creativity and innovation. After a new idea has been developed, the essential subsequent step is to evaluate its potential for successful implementation in the classroom and the broader educational context. From this viewpoint, instrumental schemas encompass individual, social, and digital or AI elements, while the formation of ideas involves both personal and social dimensions. Both students and educators develop "instrumental" schemata, suggesting that these schemata are not static but instead experience ongoing evolution within a digital or AI context, a subject that will be examined in a forthcoming study.

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